Third-order Spectral Analysis of Geolectrical Signals

Apostolos Ifantis, Vassilis Tsagaris, *Member IEEE* Vassilis Anastassopoulos, *Member IEEE* and Gerasimos-Akis Tselentis

***Abstract* — A third-order spectral analysis for the detection of non-linearities in geolectric signals is presented in this work. Specifically, the bispectrum and bicoherence function are employed for the case of the geolectrical signal acquired over the period of 1993-2002 in the area of Patras, Greece. This is an area of intense seismic activity and the possible relation between this activity and the non-linear mechanism of the geoelectrical signal is explored.**

***Keywords* — Geolectrical signal, higher order spectra.**

I. Introduction

INVESTIGATION of the changes of geoelectromagnetic signals for the purpose of earthquake prediction is carried out in United States, Russia, Japan, China, Italian, Bulgaria, and other countries. Among the different methods presented, the study of anomalies in the behavior of the geoelectromagnetic field has attracted most of the attention and over the last two decades geoelectrical measurements over a broad frequency range have been carried out. Detected signals vary in duration pattern, having specific features and spectral characteristics. There is strong evidence that anomalous changes of the geoelectromagnetic field take place prior to strong earth-quakes and great effort has been made to correlate this activity with the impending earthquakes. Geoelectromagnetic precursors to earthquakes have been reported by a number of researchers, raising hopes that prediction of damaging earthquakes might be possible [1-7].

Among the most extensive and promising reports of precursor electromagnetic signals are the observations of the Long-Term Geoelectric Potential (LTGP). An earthquake is not an instantaneous phenomenon; it is accompanied with preseismic geotectonic variations, therefore a possible correlation of the behavior of the LTGP and an oncoming earthquake is of great importance. Recently, Hayakawa [8], Uyeda [9] and Telesca [10,11] proposed a fractal analysis of Ultra Low-Frequency geoelectric data showing that strong earthquakes were preceded by a decrease of the spectral power-law exponent approaching unity.

In this work higher-order spectra are employed in order to detect nonlinearities [12] in geolectric signals. This step is being followed by locating the nonlinear components (in the Fourier domain) and quantification of these components. In particular, the bispectrum, that is the Fourier transform of the third order cumulant or moment, is employed for detection of non-linearities [13-14]. The bicoherence function, a normalized version of the bispectrum, is used to determine the energy due to nonlinear interactions. Finally, the well-known Hinich hypothesis testing is used to verify the previous results [15-16]. The data presented in this paper have been collected during a ten-year (1993-2002) independent experimental investigation at the Earthquake Prediction Section of the University of Patras Seismological Laboratory (UPSL). During the period 1993-2002 several destructive earthquakes occurred in Western Greece, a territory with the highest seismic activity in Europe.

The rest of this paper is organized as follows. In Section 2 the acquisition system along with the data are described. Section 3 provides the necessary theoretical background for data analysis. Results including a short discussion are presented in Section 4. Finally, the conclusions are drawn in Section 5.

II. LTGP Acquisition System

In this system the monitoring of the geoelectric potential difference is achieved by one set of dipoles arranged in short as well as long distances. This dipole makes use of Pb-$PbCl_2$ electrodes. The set has an electrode separation of 100m and direction E-W. The dipole is located at the outskirts of the University of Patras, in Rio, in a rather quiet countryside and is based in Pleistocene compact conglomerates. The exact geographical position of the station can be found in Figure 1(a). The signal produced by this dipole is initially directed to an electronic VAN device. Afterwards, it is directed to an A/D converter, which samples and digitizes the signal at a rate of 3 samples/min. The converter is connected to an ordinary PC for monitoring and processing the signal. This channel is also connected to a pen-recorder and a graph paper illustrates continuously the changes in the area. The obtained electro-telluric signal is transmitted via a dedicated line to the control room. The channel signal is anti-alias filtered with a 30Hz Butterworth low-pass filter sampled at 100Hz and converted to digital form with a 32-bit resolution that is depicted in Figure 1(b).

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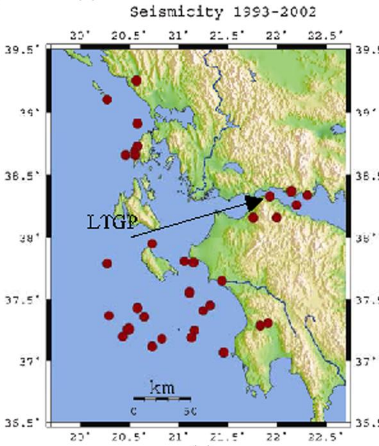
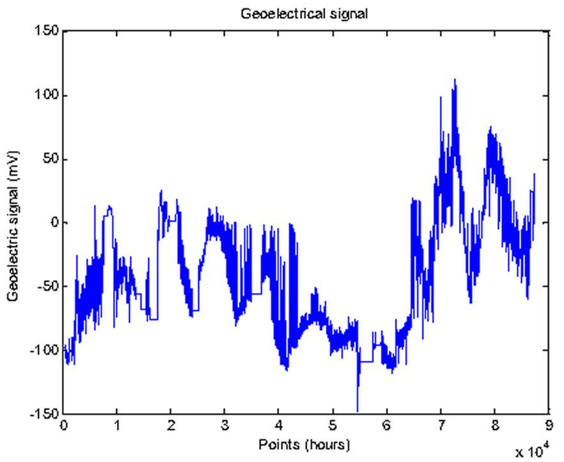
Reported electromagnetic precursor signals appear to have a wide range of time duration, amplitude level and spectral characteristics. The parameters, which are measured continuously with the above data acquisition system, are the long time variations of the geoelectrical potential. The digitization rate for observing long time variations in our station is set at 1sample/hr. Thus, approximately 86800 data (points) have been obtained during the period 1993-2002.



(a)



(b)

Figure 1: (a) Epicenters of the major earthquakes in Western Greece during the period 1993-2002 and (b) the observed geoelectrical signal for the same period.

III. THIRD-ORDER SPECTRA AND HYPOTHESIS TESTING

The bispectrum is usually defined as the Fourier transform of the third-order cumulant in accordance with the definition of the power spectrum:

$$B(\omega_1,\omega_2)=\sum_{n_1=-\infty}^{+\infty}\sum_{n_2=-\infty}^{+\infty} M_3^X(n_1,n_2)e^{-j(\omega_1 n_1+\omega_2 n_2)} \quad (1)$$

where $M_3^X(n_1,n_2)=E\{X(i)X(i+n_1)X(i+n_2)\}$. The term bifrequency is used to denote the pair (ω_1,ω_2). The estimation of the bispectrum function is based on the symmetry relationships derived from its definition in (1). For a Gaussian process the bispectrum function is zero over all frequencies, since its third-order moment is zero.

A large value of the bispectrum function in the bifrequency (ω_1,ω_1) may be caused by a large value of the power spectrum in the frequency ω_1. In [17], a normalized version of the bispectrum, the bicoherence function was introduced

$$P(\omega_1,\omega_2)=\frac{|B(\omega_1,\omega_2)|^2}{S(\omega_1)\,S(\omega_2)\,S(\omega_1+\omega_2)} \quad (2)$$

It has similar symmetry properties with the bispectrum and it can be employed to quantify the degree of non-linearity and phase coupling in the bifrequency (ω_1,ω_2).

The Gaussianity (non-skewness test) and linearity properties of the signal are examined employing the statistical test proposed by Hinich [15]:

$$S(\omega_1,\omega_2)=|b(\omega_1,\omega_2)|^2=\frac{|B(\omega_1,\omega_2)|^2}{P(\omega_1)P(\omega_2)P(\omega_1+\omega_2)} \quad (3)$$

It has been proved [15] that if a process has zero bispectrum, the statistic S is central chi-square distributed with 2K degrees of freedom. For nonzero bispectrum this statistic follows the non-central chi-square PDF with 2K degrees of freedom. We can decide with certainty that a signal is Gaussian distributed (equivalent S is central χ^2 with 2K degrees of freedom) if most of the population of the central χ^2 - PDF with 2K degrees of freedom is larger than the observed S. On the other hand, if the observed statistic S is of the largest members of a central χ^2 - PDF then we have to reject Gaussian hypothesis for the signal y(n). This certainty is expressed as the probability of false alarm P_{fa}.

If we are confident that the data are not Gaussian (non-zero bispectrum) the next step is to examine if they are nonlinear. If the data are linear we expect the bicoherence to be constant over all frequencies. This constant value can be estimated using the mean value λ of the bicoherence. The squared bicoherence in (3) is chi-squared distributed with two degrees of freedom and non-centrality parameter λ. The sample inter-quantile range R, of the squared bicoherence can be estimated and compared with the theoretical inter-quantile range for a chi-square distribution with two degrees of freedom and non-centrality parameter λ. If the estimated inter-quantile range R_{est} is much larger or far smaller than the theoretical value R_{theor}, then we should assume that the signal has undergone a nonlinear process.

IV. EXPERIMENTAL RESULTS

The first step in the spectral statistical analysis is to derive the power spectrum of the geoelectric signal. The power spectral density is estimated using the periodogram method for overlapping segments of 1024 samples and the result is depicted in Figure 2. The normalized frequency corresponds to 1024 samples that is 1024 hours in the temporal domain. The estimated power spectrum reveals a large number of peaks. The first peak is approximately in the normalized frequency 0.042 and corresponds to one-day time. The remaining observed peaks are in multiples of this basic frequency.

The next step is to employ third-order spectra in order to explore the phase coupling and the possible non-linear mechanism that produces these frequency components. The bispectrum function is estimated and plotted in Figure

3. The mean value of the bispectrum is equal to 4156 and obviously non-zero. This implies that the signal is non-Gaussian and the bicoherence function should be used to quantify the phase coupling. The bicoherence estimate is presented in Figure 4 while in Figure 5 a contour plot of the low frequencies is presented.

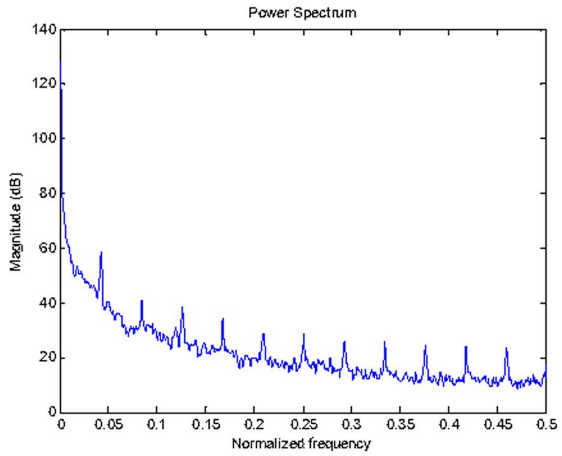


Figure 2: Power spectrum for the geoelectric signal.

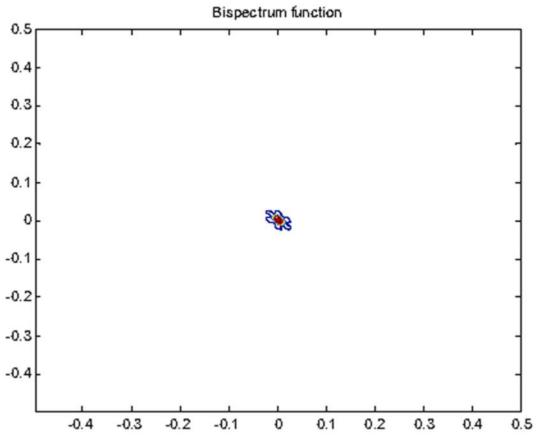


Figure 3: Bispectrum for the geoelectric signal.

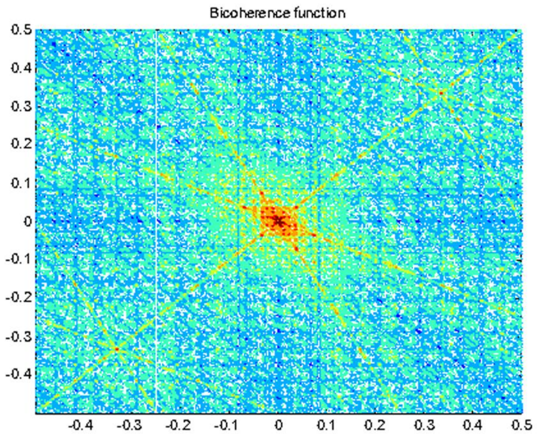


Figure 4: Bicoherence for the geoelectric signal.

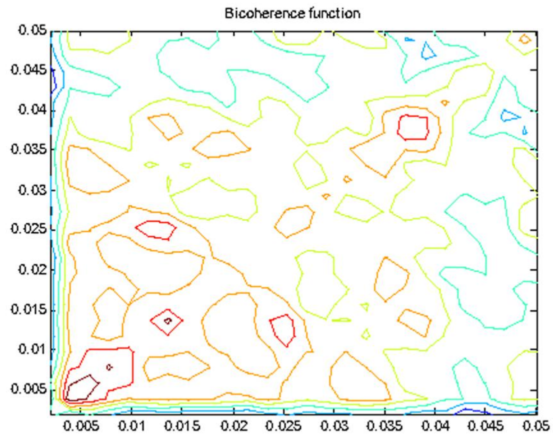


Figure 5: Low frequencies bicoherence for the geoelectric signal.

The bicoherence function reveals a significant amount of phase coupling over all frequencies as is obvious from Figure 4. Moreover, in the low frequencies, Figure 5, there is a phase coupling relationship in the bifrequency (0.025, 0.015). That is an interesting result, since there is no significant peak for those frequencies in the power spectrum. Moreover, the normalized frequency 0.042 observed in the power spectrum may be caused by phase coupling of the above frequencies.

Finally, the Hinich hypothesis testing is employed to verify the above results. The value of P_{fa} for the geolectrical signal is 0.01 denoting a non-Gaussian process. The estimated inter-quantile range R_{est} is 0.8 while the theoretical value R_{theor}, is 7.07. Based on these results we can verify that the signal has undergone a nonlinear process.

V. Conclusions and Future Work

A higher-order spectral analysis based on the bispectrum and the bicoherence function is employed in this paper. The main purpose is to detect and quantify nonlinearities that are present in geolectrical signals observed in a region with high seismic activity over a long period (1993-2002). Hinich hypothesis testing is used to verify the results obtained from the estimation of the bispectrum and the bicoherence function and reveals the presence of nonlinear mechanisms in the generation of the geolectric signals.

The nonlinear behavior of the geoelectric field probably reveals a strong connection between the dynamic mechanisms, governing the seismic and self-potential phenomena. The evolution of the Earth's crust toward the self organized criticality involves the formation of fractal structures in the fault zone. The nonlinearity of the geoelectric field indicates an increase of the degree of the persistence of the seismic process.

The authors wish to extend their work in two different aspects. Firstly, new channels will be processed in order to apply our conclusions to these channels. In this way the effect of different polarizations (East-West, North-South) in the measurement of the geolectrical signal will be examined. A different extension of this work may be the proposition of a second order Volterra filter in order to model the non-linear mechanism that generates the geolectrical signal.

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