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GRAVITY INVERSION OF A FAULT BY MARQUARDT'S METHOD

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Abstract—In this paper we discuss the solution for the inverse problem of determining the shape of a fault whose gravity anomaly is known.

A computer program in standard BASIC, based upon Marquardt's method is developed and applied to a typical gravity anomaly of a fault. The technique proved to work efficiently when tested to a number of models.

Key Words: BASIC, Faults, Geophysical anomalies, Gravity field, Marquardt's parameter, Microcomputers.

INTRODUCTION

The use of microcomputers to process and interpret geophysical data make it possible to carry out quickly complicated calculations, while being in the field, and thus speed up the interpretation of field material. This when combined with correct methodology and field operation strategy determines the success of the survey (Tselentis and Thanassoulas, 1986).

This is true particularly for the situation of gravity surveys where a vast amount of calculations are required either for the reduction of the gravity data or for the calculation of the gravity anomalies of particular geological features. One such feature is the geologic fault, and the assessment of its shape may be a target during gravity surveys.

A large number of computer algorithms have been developed for the problem of determining the size and shape of a disturbing source which gives rise to a known anomaly. The solution of such a problem (inverse), usually is achieved via optimization of the parameters, that is starting from an initial model we calculate its corresponding theoretical anomaly which is compared with the observed one. Using the residuals between the two as guides for modifying the parameters of the initial model we finally obtain the "best-fit model".

Obviously the described operation has to be done in an iterative and automatic manner otherwise it can be inaccurate and time consuming. Because the equations describing the theoretical anomalies are not linear with respect to the various parameters of the bodies the problem is actually a nonlinear least-squares problem which can be formulated as follows:

$$\sum_{i=1}^{n} (d_0^i - d_c^i)^2 = \text{minimum} \tag{1}$$

where d_0^i, d_c^i are the field and theoretical data respectively.

Minimizing Equation (1) in the least-squares we obtain the following formula which is almost the same in all the inversion problems (Figueroa, 1980).

$$d = G \cdot m \tag{2}$$

with

$$m = -(G^T G)^{-1} \cdot G^T \cdot d \tag{3}$$

where d is the measurements matrix, m the matrix containing the parameters for optimization and G the so-called Jacobian matrix whose (i,j) element is the partial derivative of the i^{th} calculated data point with respect to the j^{th} parameter.

One problem which usually is encountered is the singularity of matrix $G^T G$ in (3). This can be overcomed by Marquardt's approach writing Equation (3) as follows (Marquardt, 1963)

$$m = -(G^T G + \lambda D^2)^{-1} G^T D \tag{4}$$

where λ is a parameter known as Marquardt's parameter and D is the unit diagonal matrix

$$D_{ii}^2 = (G_T G)_{ii} \tag{5}$$

which usually is replaced by

$$D_{ii} = (G_T G)_{ii} + Q \tag{6}$$

to prevent for the possibility of being zero one of the diagonal elements of $G^T G$ in Equation (4).

Values of $\lambda = 0.4\text{E} - 3$ and $Q = 1$ have proved to be satisfactory for even the most complicated problems, so the matrix $G^T G + \lambda D^2$ is defined positively (Nash, 1978).

APPPLICATION TO THE GRAVITY FIELD OF A FAULT

A fault structure can be approximated by two semi-infinite horizontal sheets, one displaced vertically from the other. The general situation of a fault is presented in Figure 1, together with the shape of the

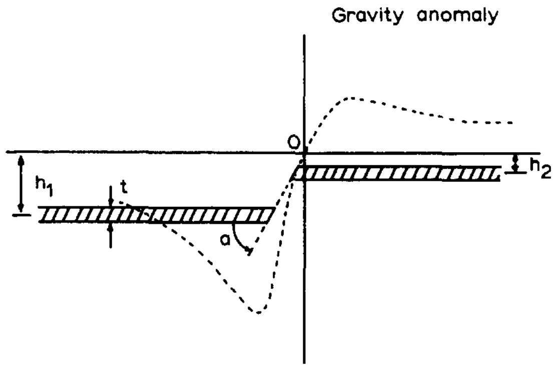

Figure 1. Fault model illustrating various parameters used in work, and shape of expected gravity anomaly.

expected anomaly which is described by the formula (e.g. Telford and others, 1979):

$$g = 2\kappa\sigma t[\pi + \tan^{-1}\{(x/h_1) + \cot a\} - \tan^{-1}\{(x/h_2) + \cot a\}] \quad (7)$$

where

σ = density contrast

t = thickness of the sheet

$h_{1,2}$ = depth of each side to the middle of the sheet

a = fault angle.

Using Equation (7), the theoretical anomaly which corresponds to a fault with $t = 500$ m, $h_1 = 6000$ m (left), $h_2 = 2000$ m, $a = 30°$, and $\sigma = 1$, is presented as a continuous line in Figure 2.

Now to solve the inverse problem, that is determine the shape and size of a fault which gives the gravity anomaly. For this purpose the program presented in the Appendix has been developed, and is based in the mathematical formulations described in the previous paragraph.

During the iterations the density constrast is kept

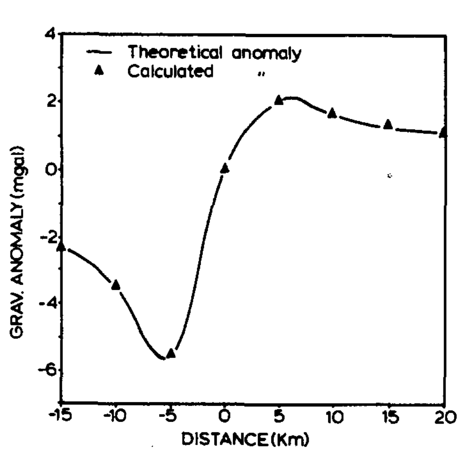

Figure 2. Theoretical gravity anomaly produced by fault with known parameters and corresponding anomaly (solid triangles) of solution given by gravity inversion program.

Table 1. Gravity anomaly for inversion

x-coordinate (m)	Gravity anomaly (mgal)
−15000	−2.24
−10000	−3.47
−5000	−5.60
0	0
5000	2.02
10000	1.61
15000	1.27
20000	1.04

as a fixed parameter, assuming that its value has been estimated previously.

The parameters which are optimized are:

(a) the thickness of the sheet,

(b) the left distance to the middle of the sheet,

(c) the right distance to the middle of the sheet, and

(d) the angle of the fault.

Starting from an initial model we calculate the anomaly $g_i(p_j)$ from Equation (7) (lines 2100–2160), the residuals (lines 480–500), and the sum of squares according to Equation (1) (lines 2400–2450).

Next line 550 switches the program to subroutine 2180 where the partial derivatives $G_{ij} = \partial g_i / \partial p_j = G(m, 4)$ are evaluated. Note that m = number of data and 4 = the number of adjustable parameters.

The initial value 0.0004 for λ (= LO) is given in line 410 whereas line 410 assigns the value of Q in Equation (6), (= PO) to 1. The quantity m defined in Equation (4) is evaluated as a (4×4) matrix from lines 620 to 840.

After each iteration, the new sum of squares Z is compared with the old sum S, (line 1120), and if $S > Z$ the parameter is reduced by a factor of 0.4 (line 1220) and a new iteration begins. If after an iteration $S < = Z$, the parameter is increased by a factor of 10 (line 1240) and a new set of updated parameters is evaluated.

Anytime a new iteration is terminated lines 1260–1340 inform us whether the process is going in the right direction and provide the facility to stop the program when a satisfactory match between observed and calculated anomalies is obtained.

To test the program, the theoretical anomaly of Figure 2 is digitized every 5000 m (Table 1), and a "bad" initial model with parameters $h_1 = 3000$ m, $h_2 = 1600$ m, $t = 700$ m, and $a = 30°$ is entered.

The program after a number of iterations, brings the sum of squares from 18 to 2.5E − 4 which corresponds to a model with parameters given in Table 2, and which are close to the actual ones. The corresponding values of the gravity field generated by the solution are ploted as triangles in Figure 2.

An important factor which influences the con-

Table 2. Parameters of obtained solution

Thickness = 500 m
Angle a = 60°
h_1 = 5780 m
h_2 = 1753 m

vergence speed and accuracy of the algorithm is obviously the initial model. It is critical to take into account all the known or suspected geological features of the fault in order to obtain a reasonable final model in less time.

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APPENDIX

Computer Program in IBM(pc) BASIC

```
100 REM ###############################
110 REM #  FAULT  MODELING      #
120 REM ########################
150 REM by C.Thanassoulas, G-A.Tselentis, K.Dimitriadis
190 INPUT "GIVE No OF MEASUREMENTS";M
200 DIM P(4,1),F(M,1),C(M),O(M),X(M)
210 DIM J(M,4),T(4,M),P$(5),E(4,2)
220 DIM V(4,4),P1(4,1),Q(4,1)
230 DIM S(1),Z(1),P8(1,4),BB(4,4)
235 INPUT"COORDINATES IN METERS OR FEET";CO$
240 PRINT"GRAVITY VALUES IN MILLIGALS"
250 PRINT"COORDINATES IN ";CO$
260 FOR I=1 TO M
270 PRINT I:INPUT "COORD";X(I)
290 INPUT "ANOMALY";O(I)
300 NEXT I
310 P$(1)="BED THICKNESS:"
320 P$(2)="DIPPING ANGLE:"
330 P$(3)="LEFT DIST. TO THE TOP:"
340 P$(4)="RIGHT DIST. TO THE TOP:"
350 INPUT"BED THICKNESS";P(1,1)
360 INPUT "DIPPING ANGLE";P(2,1)
370 P(2,1)=P(2,1)*3.141593/180
380 INPUT"LEFT DIST. TO THE TOP ";P(3,1)
390 INPUT"RIGHT DIST TO THE TOP ";P(4,1)
400 INPUT"DENSITY CONTRAST ";D0
410 L0=.0004:G0=.00667
415 IF CO$="F" THEN G0=.002035
420 GOSUB 2100 :REM THEORETICAL ANOMALY
430 REM *** TRANSPOSE ***
440 FOR I=1 TO 4
450 P8(1,I)=P(I,1)
460 NEXT I
480 FOR I=1 TO M
490 F(I,1)=C(I)-O(I)
500 NEXT I
510 REM
520 REM *** SQUARES SUM ***
530 GOSUB 2400
540 S(1)=SU
550 GOSUB 2180 :REM JACOBIAN
560 P0=1
570 FOR I=1 TO M
580 FOR AA=1 TO 4
590 T(AA.I)=J(I,AA)
600 NEXT AA
610 NEXT I
620 REM *** TXF ***
630 FOR I=1 TO 4
640 FOR J=1 TO 1
650 P1(I,J)=0!
660 FOR K=1 TO M
670 P1(I,J)=P1(I,J)+T(I,K)*F(K,J)
680 NEXT K
690 NEXT J
700 NEXT I
```

```
710 REM *** TXJ ***
720 FOR I=1 TO 4
730 FOR JJ=1 TO 4
740 V(I,JJ)=0!
750 FOR K=1 TO M
760 V(I,JJ)=V(I,JJ)+T(I,K)*J(K,JJ)
770 NEXT K
780 NEXT JJ
790 NEXT I
810 FOR I=1 TO 4
820 V(I,I)=V(I,I)*(1+LO)+LO*PO
830 NEXT I
840 REM
850 N=4
860 GOSUB 1600
870 FOR I=1 TO N
880 FOR J=1 TO N
890 V(I,J)=-BB(I,J)
900 NEXT J
910 NEXT I
920 REM *** VXP1***
930 FOR I=1 TO 4
940 FOR J=1 TO 1
950 Q(I,J)=0!
960 FOR K=1 TO 4
970 Q(I,J)=Q(I,J)+V(I,K)*P1(K,J)
980 NEXT K
990 NEXT J
1000 NEXT I
1020 FOR I=1 TO 4
1030 P(I,1)=P(I,1)+Q(I,1)
1040 NEXT I
1050 GOSUB 2100
1060 FOR I=1 TO M
1070 F(I,1)=C(I)-O(I)
1080 NEXT I
1090 REM *** WXF ***
1100 GOSUB 2400
1110 Z(1)=SU
1120 IF Z(1)>=S(1) THEN 1240
1125 TC=Z(1)-S(1)
1130 IF ABS(TC) <.00001 AND Z(1) <.1 THEN 1400
1140 FOR I=1 TO 4
1150 P8(1,I)=P(I,1)
1160 NEXT I
1170 GOSUB 2310
1180 PRINT "THE LEAST SQUARE SUMS"
1190 PRINT"FOR THE OLD AND NEW ITERATIONS ARE"
1200 PRINT S(1);"----";Z(1)
1210 PRINT"---------------------------------------"
1220 LO=LO*.4
1230 GOTO 510
1240 LO=LO*10
1250 PRINT
1260 PRINT"THE OLD SUM IS:";S(1)
1270 PRINT"THE NEW SUM IS:";Z(1)
1280 PRINT"THE ACTUAL PTRS ARE GOING IN THE WRONG DIRECTION"
1300 PRINT "PRESS ANY KEY TO GET THE NEW CORRECTED ORIENTATION'
1320 PRINT"OR PRESS F TO GET THE FINAL VALUES"
1330 INPUT  H$:IF H$="" THEN 1330
1340 IF H$="F" THEN 1400
1350 PRINT"O.K."
1360 FOR I=1 TO 4
1370 P(I,1)=P8(1,I)
1380 NEXT I
1390 GOTO 710
1400 PRINT" OBSERVED ANOMALY "
1410 FOR I=1 TO M
1420 PRINT I,O(I)
1430 NEXT I
1440 PRINT
1450 PRINT" CALCULATED ANOMALY"
1460 FOR I=1 TO M
1470 PRINT I,C(I)
1480 NEXT I
1490 PRINT "RESIDUALS"
1500 FOR I=1 TO M
1510 PRINT I,F(I,1)
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```
1520 NEXT I
1530 PRINT "THICKNESS";P8(1,1)
1540 PRINT"ANGLE  ";P8(1,2)*180/3.141593
1550 PRINT"LEFT DIST. TO THE TOP":Z3
1560 PRINT"RIGHT DIST. TO THE TOP";Z1
1570 PRINT"DENSITY CONTRAST ";D0
1580 PRINT"###########################################################"
1590 END
1600 REM
1610 REM
1620 FOR I=1 TO N
1630 K(I)=0
1640 L(I)=0
1650 NEXT I
1660 MR=1
1670 FOR I=1 TO N
1680 FOR J=1 TO N
1690 BB(I,J)=V(I,J)
1700 NEXT J
1710 NEXT I
1720 FOR H=1  TO N
1730 TM=0!
1740 FOR I=1 TO N
1750 IF L(I) <>0 THEN 1830
1760 FOR J=1 TO N
1770 IF K(J)<>0 THEN 1820
1780 IF ABS(BB(I,J))<=TM THEN 1820
1790 TM=ABS(BB(I,J))
1800 LM=I
1810 KP=J
1820 NEXT J
1830 NEXT I
1840 IF TM=0! THEN 2080
1850 L(LM)=KP
1860 K(KP)=LM
1870 BO=BB(LM,KP)
1880 FOR I=1 TO N
1890 IF I=LM THEN 1940
1900 FOR J=1 TO N
1910 IF J=KP THEN 1930
1920 BB(I,J)=BB(I,J)-BB(I,KP)*BB(LM,J)/BO
1930 NEXT J
1940 NEXT I
1950 FOR I=1 TO N
1960 BB(LM,I)=-BB(LM,I)/BO
1970 IF I=KP THEN BB(LM,I)=1/BO
1980 IF I=LM THEN 2000
1990 BB(I,KP)=BB(I,KP)/BO
2000 NEXT I
2010 NEXT H
2020 FOR I=1 TO N
2030 FOR J=1 TO N
2040 BB(I,J)=BB(L(I),K(J))
2050 NEXT J
2060 NEXT I
2070 RETURN
2080 MR=0
2090 RETURN
2100 REM
2110 FOR I=1 TO M
2120 C1=(X(I)/P(4,1))+(1/TAN(P(2,1)))
2130 C2=(X(I)/P(3,1))+(1/TAN(P(2,1)))
2140 C3=ATN(C1):C4=ATN(C2)
2150 C(I)=2*G0*D0*P(1,1)*(C3-C4)
2160 NEXT I
2170 RETURN
2180 REM JACOBIAN
2190 SW=SIN(P(2,1))
2200 DD=(2*G0*D0*P(1,1)/(SW^2))
2210 FOR I=1 TO M
2220 C1=(X(I)/P(4,1))+(1/TAN(P(2,1)))
2230 C2=(X(I)/P(3,1))+(1/TAN(P(2,1)))
2240 J(I,1)=C(I)/P(1,1)
2250 V2=1/(C2^2+1):V1=1/(C1^2+1)
2260 J(I,2)=DD*(V2-V1)
2270 J(I,3)=2*G0*D0*P(1,1)*V2*(P(3,1)^(-2))*X(I)
2280 J(I,4)=(-2)*G0*D0*P(1.1)*V1*(P(4,1)^(-2))*X(I)
2290 NEXT I
```

```
2300 RETURN
2310 REM
2320 PRINT"THE NEW PARAMETERS ARE NOW"
2330 Z3=P(3,1)-(P(1,1)/2)
2340 Z1=P(4,1)-(P(1,1)/2)
2350 PRINT P$(1);P(1,1)
2360 PRINT P$(2);P(2,1)*180/3.141593
2370 PRINT P$(3);Z3
2380 PRINT P$(4);Z1
2390 RETURN
2400 REM
2410 SU=0!
2420 FOR I=1 TO M
2430 SU=SU+F(I,1)^2
2440 NEXT I
2450 RETURN
```