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BAYESIAN PROBABILISTIC PREDICTION OF STRONG EARTHQUAKES IN THE MAIN SEISMOGENIC ZONES OF GREECE

Summary. The probabilities of occurence of moderate and strong earthquakes are computed in the main seismogenic zones of Greece using Bayesian statistics.

The seismicity in each seismic zone is obtained in two steps. Firstly, the occurrence of events is considered independently of the magnitudes, and then the probability distribution of the magnitudes is introduced.

Finally, a seismic hazard map is compiled which shows the seismic zones in Greece according to the expected year of occurrence of moderate to strong earthquakes.

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1. Introduction

Bayesian probability theory, although controversional, makes the formulation of inferences from data straightforward and allows the investigation of problems that would be otherwise unapproachable. What Baye's theorem does is to describe the way in which we learn from experience, making clear the fact that probability assignments define one's degree of belief, and are always conditional on the state of information. Baye's theory is generally considered one of subjective probability or personal probability as it is sometimes called (Savage, 1961).

Benjamin (1968) assuming a Poisson distribution, was the first to develop a Bayesian distribution of earthquake occurrences. A similar application based on different distributions was later presented by Chou et al., 1971. Esteva (1969), and Lomnitz (1969), used a similar approach to study the seismicity of Mexico and Chile, respectively. Campbell (1982, 1983) proposed a Bayesian extreme value distribution of earthquake occurrence to evaluate the seismic hazard along the San Jacinto Fault. The same procedure has been applied by Stavrakakis and Tselentis (1986) in order to predict the maximum expected earthquake magnitude along the Hellenic arc.

Recently, Ferraes (1985, 1986) used a Bayesian analysis to predict the inter - arrival times for strong earthquakes along the western and eastern Hellenic arc as well as for strong earthquakes felt in Mexico City.

In the present work, we have applied Bayesian statistics in conjuction with the conditional Poisson model in order to compute probabilities of earthquake occurrence and magnitude distribution of moderate and strong earthquakes in the main seismogenic zones of Greece as defined by Papazachos (1980).

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2. Bayesian model of earthquake occurrence

It is assumed that the earthquake occurrence in each seismogenic zone forms a Poisson process with mean rate of occurrence independent of magnitude. Considering all strong earthquakes with magnitude greater than M a distribution of the number of occurrences can be obtained for a given period of time.

In the most general form, the conditional Poisson law is written as

$$P\ (\eta/\lambda)\ =\ \frac{e^{-\lambda t}\ (\lambda\ t)^{\eta}}{\eta'} \tag{1}$$

with $t > 0$, n integer, where $P\ (n/\lambda)$ is the probability of having n events in time period t, λ is the mean rate of occurrence per unit time, and $\mu_N = \lambda$, $\sigma_N^2 = \lambda$ the mean and the variance of the distribution, respectively.

In classical statistics the crucial problem is to determine the mean rate of the Poisson distribution. Usually, this parameter is considered to be deterministic and is estimated from historical frequency data. However, there is still uncertainty about the parameter λ, and therefore it is treated as a random variable.

Let $f'\ (\lambda)$, $L\ (\lambda)$ be the prior distribution function and the sample likelihood function on λ, respectively. The posterior probability $f''\ (\lambda)$ is then obtained by using Bayes' theorem

$$f''\ (\lambda)\ =\ N\ L\ (\lambda)\ f'\ (\lambda) \tag{2}$$

where N is a normalizing constant. In the following we seek the prior distribution, sample likelihood function, and the posterior distribution on λ following Mortgat and Shah (1979).

2.1. Prior distribution on λ

The prior distribution on λ is chosen as the gamma distribution with parameters λ' and v'. This choice does not introduce any limitations (Raiffa and Schlaifer, 1961) since the gamma distribution can fit a large variety of shapes. The parameters λ' and v' are obtained from the data and the prior distribution can be written

$$f'\ (\lambda)\ =\ \frac{\lambda'\ (\lambda'\ \lambda)\ e^{-\lambda'\lambda}}{\Gamma\ (v')} \tag{3}$$

where

$$\Gamma\ (v')\ =\ \int_o^{\infty}\ e^{-u}\ u^{\nu'-1}\ d\ u$$

$$\mu\ =\ v'\ /\ \lambda'$$

$$\sigma^2\ =\ v'\ /\ \lambda'^2$$

2.2. Sample likelihood function on λ

For any seismogenic region in Greece, the available data indicates that in the past T years N strong earthquakes with magnitude greater than 5.5. have occurred. This is basic information for the construction of the sample likelihood function. Since the earthquake occurrence is assumed to be a Poisson process, the sample likelihood function on λ is given by

$$L\ (\lambda/N,\ T)\ =\ \frac{e^{-\lambda T}\ (\lambda T)^{N}}{N!} \tag{4}$$

2.3 Posterior distribution on λ

Combining (3) and (4) by means of Baye's theorem (2), the posterior distribution on λ can be written as

$$f''(\lambda) = N \frac{e^{-\lambda T} (\lambda T)^N}{N!} \cdot \frac{\lambda' (\lambda' \lambda)^{v'-1} e^{-\lambda'\lambda}}{\Gamma (v')} \tag{5}$$

Since $f''(\lambda)$ is a probability distribution, we have that

$$\int_0^\infty f''(\lambda)\, d\lambda = 1.0$$

and the normalizing constant N can be estimated. Rearranging (5), the posterior distribution on λ can be written as

$$f''(\lambda) = \frac{\lambda'' (\lambda'' \lambda)\, e^{-\lambda''\lambda}}{\Gamma (v'')} \tag{6}$$

with $\mu = v''/\lambda''$, $\sigma^2 = v''/\lambda''^2$ where:

$$\lambda'' = \lambda' + T$$

$$v'' = v' + N$$

Equation (6) indicates that the posterior distribution on λ is also of the gamma type.

Finally, the unconditional distribution on the number of earthquake occurrence can be obtained by using (6) with (1) and integrating over all 's. Hence, the probability of having n events int he next t years, irrespective of the mean rate λ, is

$$P(\eta) = \int_0^\infty P(\eta/\lambda)\, \lambda\, d\lambda$$

$$= \int_0^\infty P(\eta/\lambda)\, f''(\lambda)\, d\lambda$$

$$= \int_0^\infty \frac{e^{-\lambda t} (\lambda t)^\eta}{\eta!} \cdot \frac{\lambda'' (\lambda'' \lambda)^{v''-1} e^{-\lambda''\lambda}}{\Gamma (v'')}\, d\lambda$$

$$= \int_0^\infty \frac{e^{-\lambda (t+\lambda'')}\, t^\eta\, \lambda''^{v''}\, \lambda^{\eta+v''-1}}{\eta!\, \Gamma (v'')}\, d\lambda$$

$$= \frac{\Gamma (\eta + \nu'')}{\eta!\, \Gamma (\nu'')} \cdot \frac{t^\eta \lambda''^{\nu''}}{(t + \lambda'')^{\eta+v''}} \tag{7}$$

In the case that v'' has integer values, eq. (7) can be written

$$P(\eta) = \frac{(\eta + v'' - 1)!}{\eta!\, (\nu'' - 1)!} \cdot \frac{t^\eta \lambda''^{\nu''}}{(t + \lambda'')^{\eta+\nu''}}$$

$$= \frac{\Gamma\,(\eta\,+\,\nu")}{\eta\,!\,\Gamma\,(\nu")}\cdot\frac{t^{\eta}\,\lambda"^{\nu"}}{(t\,+\,\lambda")^{\eta+\nu"}} \tag{8}$$

Setting $\lambda" = T$ and $v" = N+1$, the above equation can be written in a simplified form

$$P\,(\eta) = \frac{\Gamma\,(\eta\,+\,N\,+\,1)}{\eta\,!\,\Gamma\,(N\,+\,1)}\cdot\frac{t^{\eta}\,T^{N+1}}{(t\,+\,T)^{\eta+N+1}} \tag{9}$$

Equation (8) gives the Bayesian probability of occurrence of the number of earthquakes above a predetermined bound M in the next t years, and eq. (8) the same probability in the same time period, given that N earthquakes have occurred in the past T years.

3. Bayesian model of distribution of magnitude

Up to this point, the seismicity of each seismic source has been defined, only by the distribution of the number of earthquakes that this source may generate in a given time period t. The next step is to determine the probability that there will be 0, 1, 2, ..., n events of any given magnitude, based on the fact that n earthquakes will occur in future time t.

This probability is obtained by applying Bayesian statistics since the classical Poisson distribution gives only the probability of occurrence of n events with magnitude greater than a predetermined bound in future time t.

Suppose that there are r possible magnitudes, and p_i is the probability of an event with magnitudes m_i, $i = 1, \ldots r$. Without any other information, the prior distribution p_i would be

$$f'\,(p_1,\,p_2,\,\ldots,\,p_r) = \begin{cases} K & \text{if } \sum p_i = 1 \\ 0 & \text{otherwise} \end{cases} \tag{10}$$

where $K = (r-1)!/\sqrt{r}$.

The usual form of the seismological data indicates that among the N earthquakes which have occurred in a seismic zone, Xi were of magnitude Mi. This information is used to construct the posterior distribution of p_i given by

$$f"\,(p_1,\,\ldots,\,p_r\,/\,X_r,\,N) = \frac{(N\,+\,r\,-\,1)\,!}{X_1\,!\,X_2\,!\,\ldots\,X_r\,!}\cdot P_1^{X_1}\,P_2^{X_2}\,\ldots\,P_r^{X_r}\,/\,\sqrt{r} \tag{11}$$

with $Xi = N$ and $\Sigma\, p_i = 1$.

The unconditional (marginal) distribution of pi is expressed as

$$f"\,(p_i) = \frac{(N\,+\,r\,-\,1)\,!}{X_i\,!\,(N\,+\,r\,-\,X_i\,-\,2)\,!}\,p_i^{X_i}\,(1\,-\,P_i)^{N\,-\,X_i\,+\,r\,-\,2} \tag{12}$$

and the expected value of p_i is given by

$$P_i = \int_0^1 P_i\,f"\,(P_i)\,d\,p_i = \frac{X_i\,+\,1}{N\,+\,r} \tag{13}$$

where X_i the number of earthquakes of magnitude M_i. N is the total number of earthquakes that have occurred in a seismic zone and r is the number of different magnitudes corresponding to the seismic zone.

4. Application of the Bayesian model to Greece

The Bayesian model, as described above, has been applied to the main seismogenic zones of Greece in order to obtain the probability distribution of earthquakes of occurrence with $M > 5.5$ and the probability distribution of earthquake magnitudes.

Fig. 1 shows the Aegean and surrounding area (34°N-42°N, 19°E-29°E) which has been divided into 19 seismic zones (Papazachos, 1980) on the basis of seismotectonic criteria, such as seismic rates, focal mechanisms, etc. For each seismic zone, only events with $M > 5.5$ have been taken from the earthquake catalogues prepared by Makropoulos and Burton (1981) and by Comninakis and Papazachos (1986). For seismic zone 2, only earthquakes with $M > 5.8$ have been considered because of its high seismicity.

Table 1, summarizes the number of earthquakes corresponding to each seismic zone as well as the number of events within intervals of magnitude $M = 0.4$. Furumoto (1966) has pointed out that the scatter is reduced by using intervals of magnitude of about 0.5 and for purposes of prediction there is no loss of information.

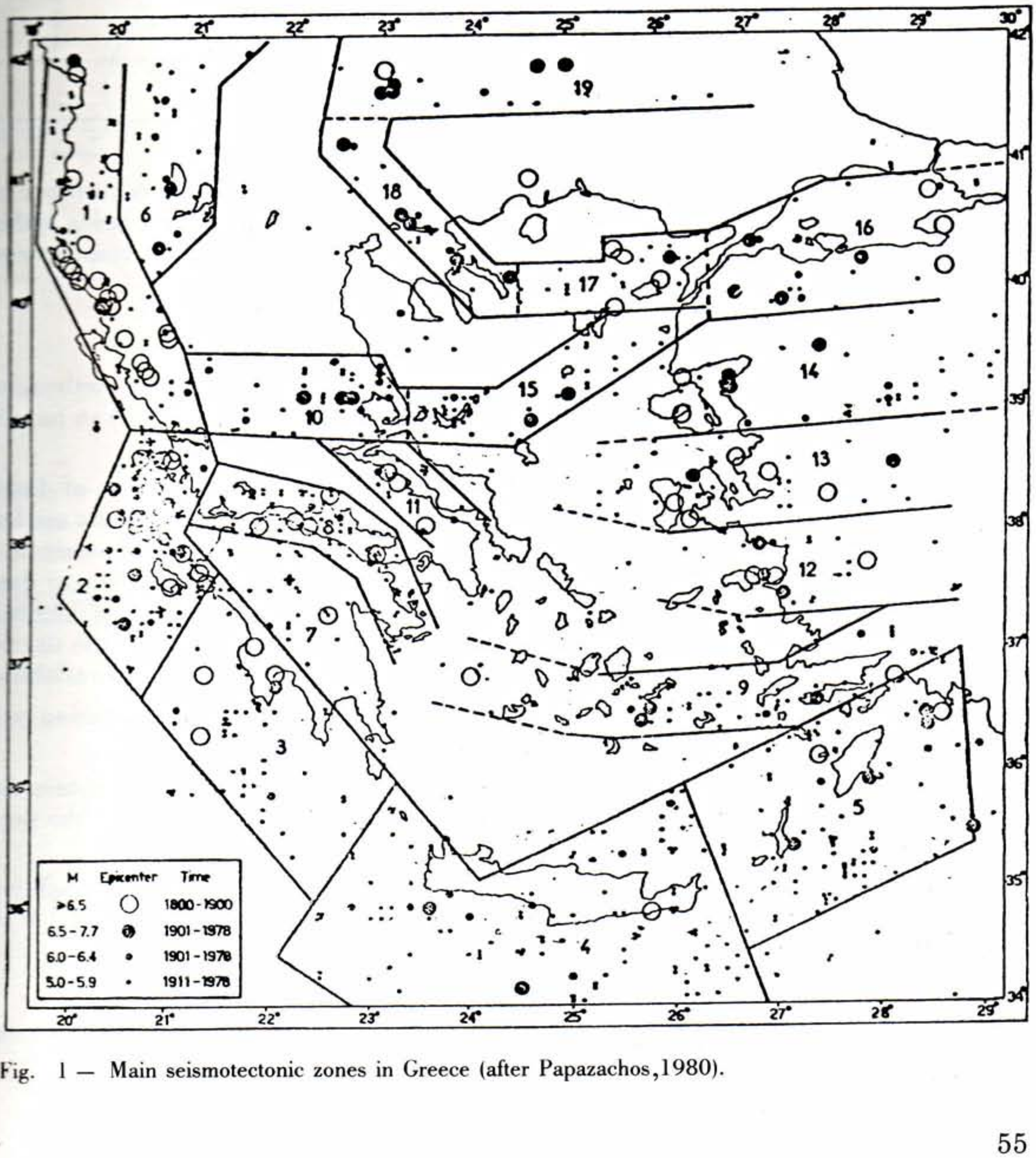

Fig. 1 — Main seismotectonic zones in Greece (after Papazachos, 1980).

Table 1 — Information on number of earthquakes with different interval magnitudes for each seismogenic zone in the area of Greece.

Zone	No of events with M 〉 5.5	No of classes M (r)	M = 5.5-6.0 X_1	M = 6.1-6.5 X_2	M =	M= 6.6-7.0 X_3
1	27	3	23	2	2	0
3	11	3	8	1	2	0
4	39	3	11	7	1	0
5	16	4	13	1	1	1
6	17	3	13	3	1	0
7	7	2	6	1	0	0
8	18	3	13	3	2	1
9	10	4	6	2	1	0
10	19	3	12	5	2	0
11	7	2	5	2	0	0
12	11	3	8	1	2	0
13	12	3	9	1	2	0
14	11	3	8	1	2	0
15	8	4	5	0	1	2
16	15	4	8	2	3	2
17	7	3	5	1	1	0
18	12	4	7	2	2	1
19	10	5	6	2	0	1
with			M=5.8-6.3	M=6.4-6.9	M=7.0-7.5	
2 M 〉 5.8	27	3	19	6	2	—

For each seismic zone, the Bayesian probability distribution of the occurrence of one earthquake with $M \rangle 5.5$ in the next t years, and the probability of it being within a fixed magnitude range have been calculated according to (9) and (13), respectively. The results are shown in a graphical form in Figs. 2 to 5.

5. Discussion

In this paper we have shown how the Bayesian model provides a rational methodology for evaluating future seismicity. The structure of the model is such that it can handle any quality and quantity of information in a consistent manner.

The main problem in earthquake statistics is to determine the parameters of distribution for the earthquake occurrence and magnitude. Usually, these parameters are based on the observed frequencies alone, and there is no possibility that the actual parameters could take other values. Obviously, this is not true, especially in the case where data is scare. One could not say, for example, that the mean rate, λ, of the Poisson distribution is zero if no events had been observed in the past ten years. Because of the uncertainty in the estimation of the parameters they should be treated as random variables.

The difference between the Poisson model with mean rate, λ, and the Bayesian model, can be demonstrated by considering the following hypothetical case.

Assume $N/T = 0.1$, and a time period $t = 10$ years, Then, from (9) we obtain the probability of no occurrences and the probability of at least one occurrence in the next 10 years. The results obtained are listed in Table 2.

It should be emphasized that the Poisson model always gives a smaller probability of at least one occurrence. This is because the mean rate for the Poisson model $\lambda = N/T$ which is smaller than the expected mean rate λ for the Bayesian model

$$\lambda = \int_0^{\infty} \lambda f''(\lambda/N, T)\, d\lambda = (N + \perp)/T$$

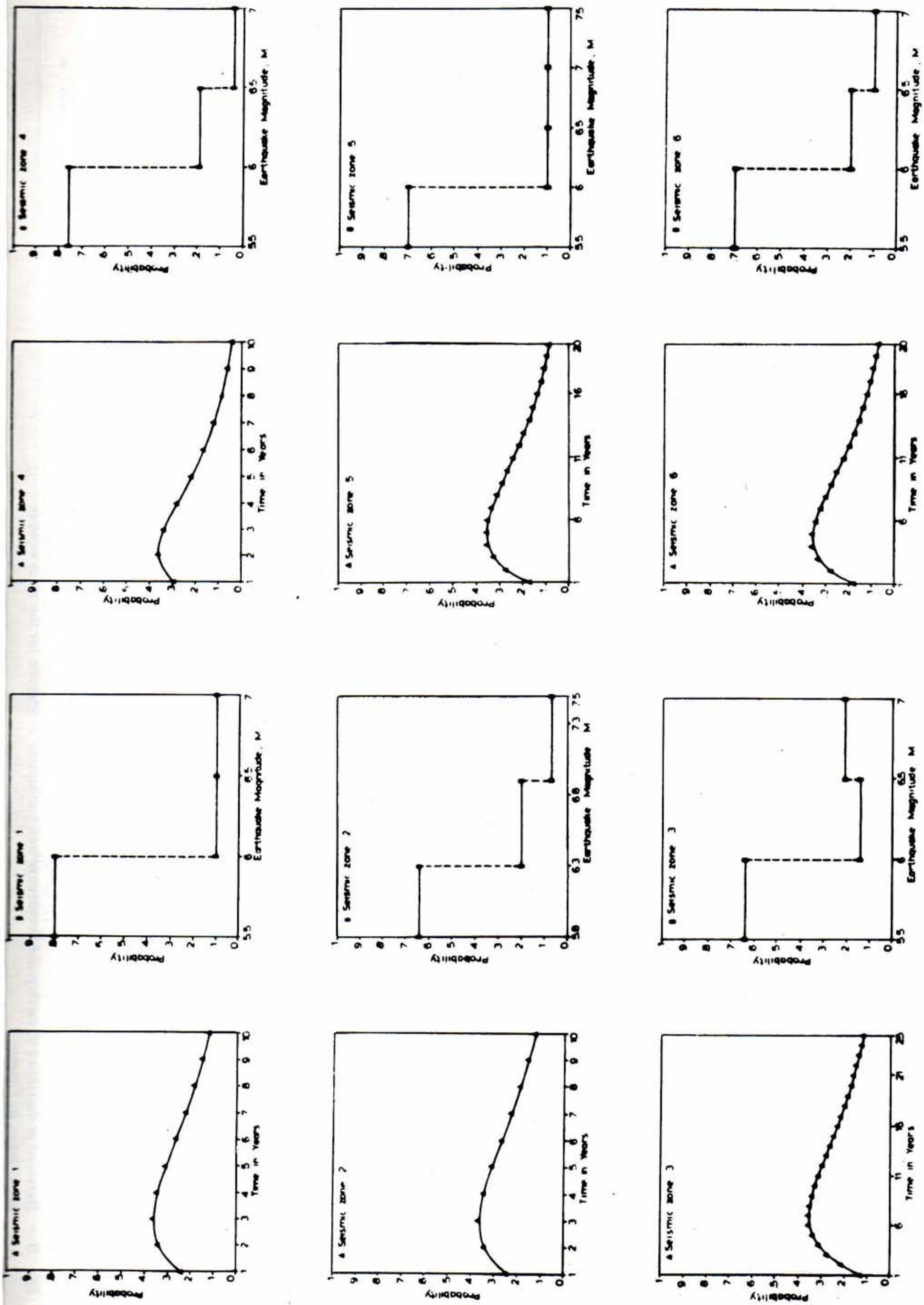

Fig. 2 — Bayesian probabilities of earthquake occurrence and earthquake magnitude for the main seismogenic zones in Greece.

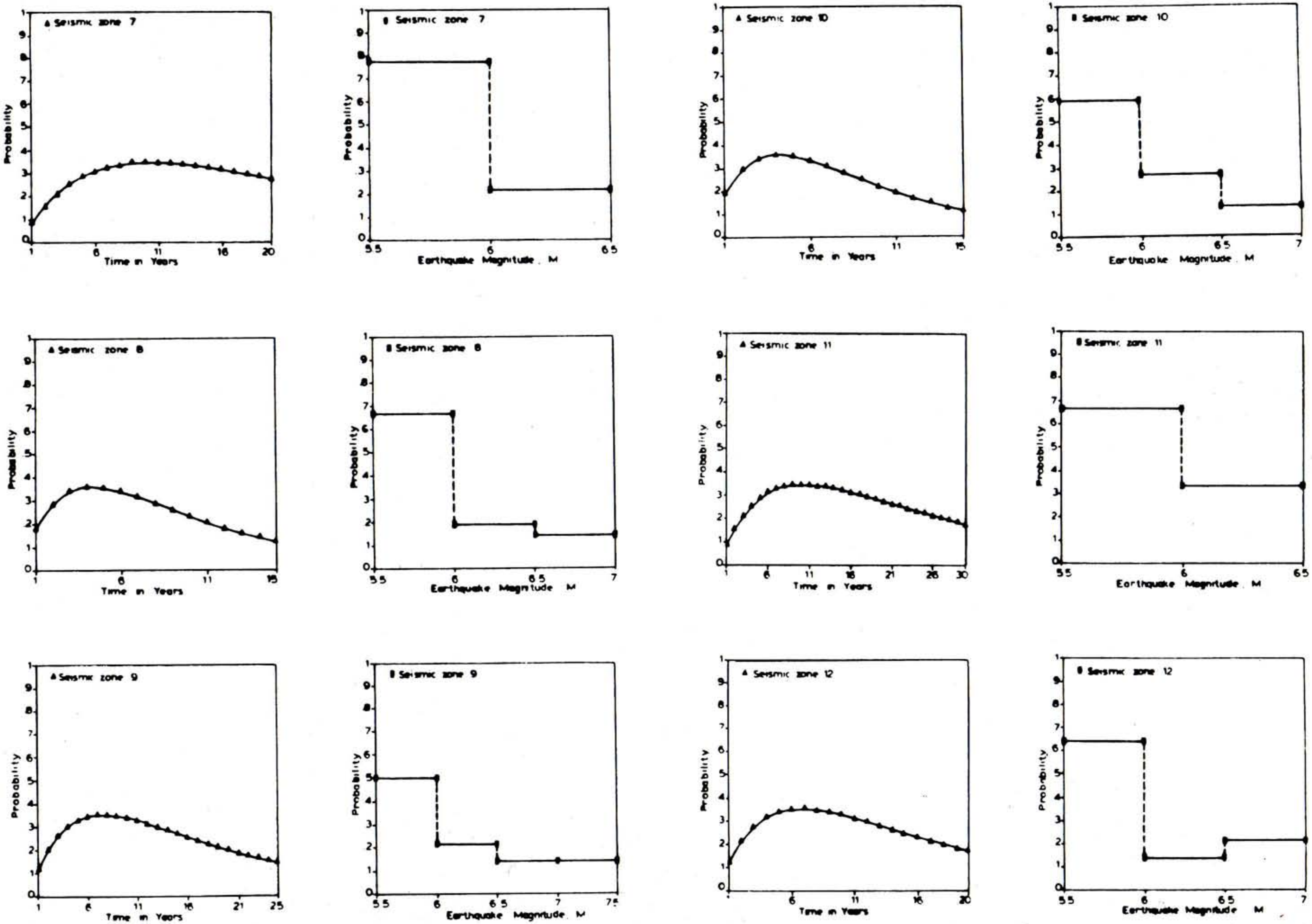

Fig. 3 — Bayesian probabilities of earthquake occurrence and earthquake magnitude for the main seismogenic zones in Greece.

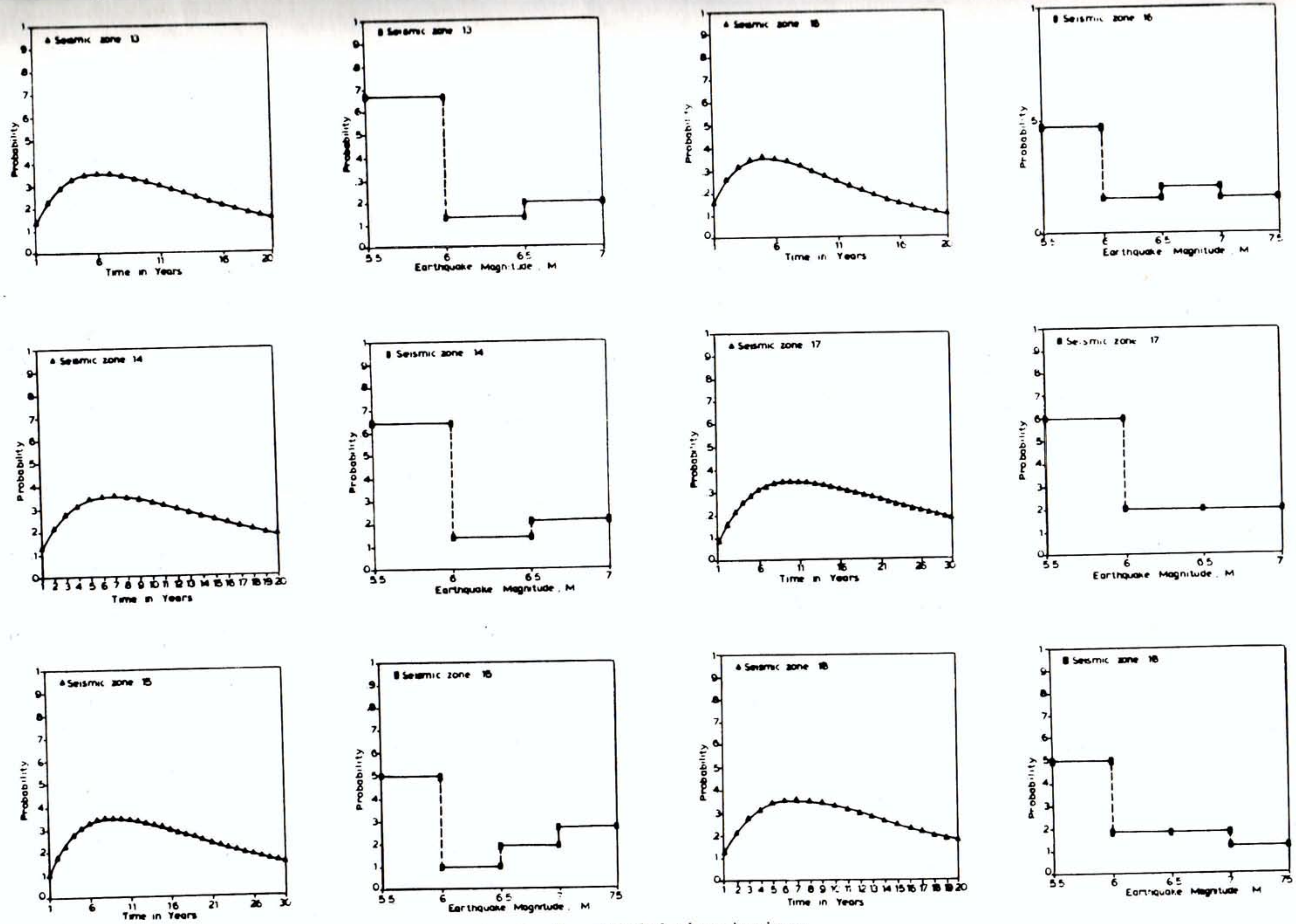

Fig. 4 — Bayesian probabilities of earthquake occurrence and earthquake magnitude for the main seismogenic zones in Greece.

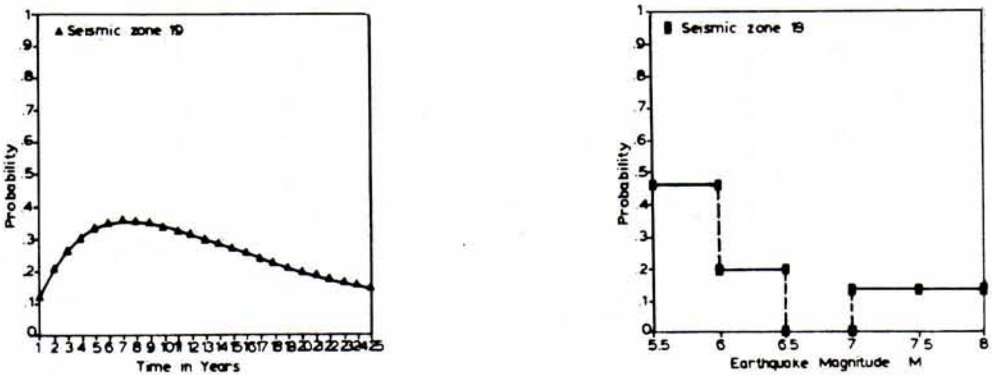

Fig. 5 — Bayesian probabilities of earthquake occurrence and earthquake magnitude for the main seismogenic zones in Greece.

Table 2 — Comparison between Bayesian and Poisson models.

MODEL		PN (n=0)	PN (n〉0)
	Poisson, λ = 0.1	**0.368**	**0.632**
Bayes	T = 10, N = 1	0.250	0.750
	T = 100, N = 10	0.351	0.649
	T = 1000, N = 100	0.366	0.634

Thus, the Poisson model will in general underestimate the expected future seismicity.

Our results can not be compared directly with previous works because most long term prediction studies are mainly concerned with the west Hellenic arc, which almost coincides with the seismogenic zone 3 in Fig. 1.

Within this zone, Papazachos (1980) has observed a drop of the seismicity rate since 1961 and suggests that a strong earthquake *(M 〉 7.0)* will probably occur there, in the next ten years or so. Wyss and Baer (1981) based on the seismicity patterns along the Hellenic arc suggested that large earthquakes should be expected to occur between 1980 and 1990 near Anticithira and Carpathos (zones 3 and 5 respectively in Fig. 1). Papazachos and Comninakis (1982) observed that seismic quiescence is a precursory anomaly, and since the western Hellenic arc has been quiescent since 1967, suggested that an earthquake of $M > 7.0$ may be expected in 1992.

According to the Bayesian probability distribution, we expect a strong earthquake in the western Hellenic arc between 1989 and 1991, of magnitude range 5.5-6.0, 6.1-6.5 or 6.6-7.0 with probabilities 0.64, 0.14 and 0.21, respectively.

The probability of occurrence one earthquake within the magnitude range *Mi* for each seismic zone is shown in Fig. 6. As we see, the highest probability corresponds to seismic zones 1, 4 and 7 for earthquake magnitudes between 5.5-6.0, to seismic zone 11 for $M = 6.1\text{-}6.5$ to 3.12 and 16 for $M = 6.6\text{-}7.0$ and to seismic zones 3 and 15 for $M = 7.1\text{-}7.5$.

The expected year of earthquake occurrence with $M > 5.5$ for each seismic zone is shown in Fig. 7. The shortest time corresponds to 1985 for seismic zone 4 where an earthquake with $M = 5.5$ occurred on September 27, 1985 (Comninankis and Papazachos 1986). Also in seismic zone 1, an earthquake of $M_L = 5.1$ occurred on Dec. 17, 1986 according to preliminary seismological Bulletins of the National Observatory of Athens.

The longest expected time of earthquake occurrence with $M > 5.5$ corresponds to

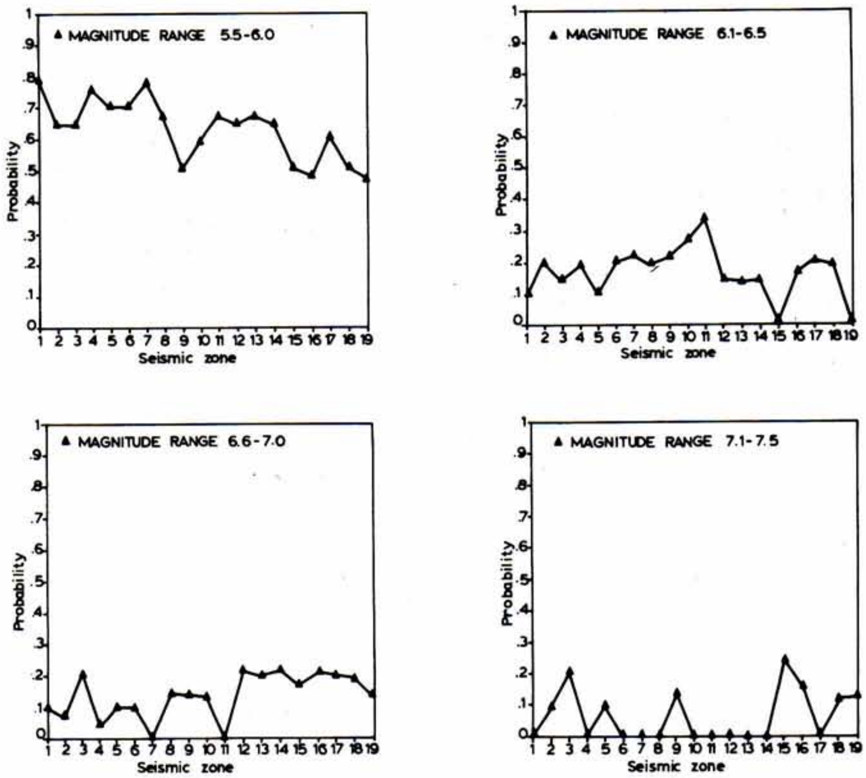

Fig. 6 — Probability of earthquake occurrence within the magnitude range M_j for each seismogenic zone in Greece.

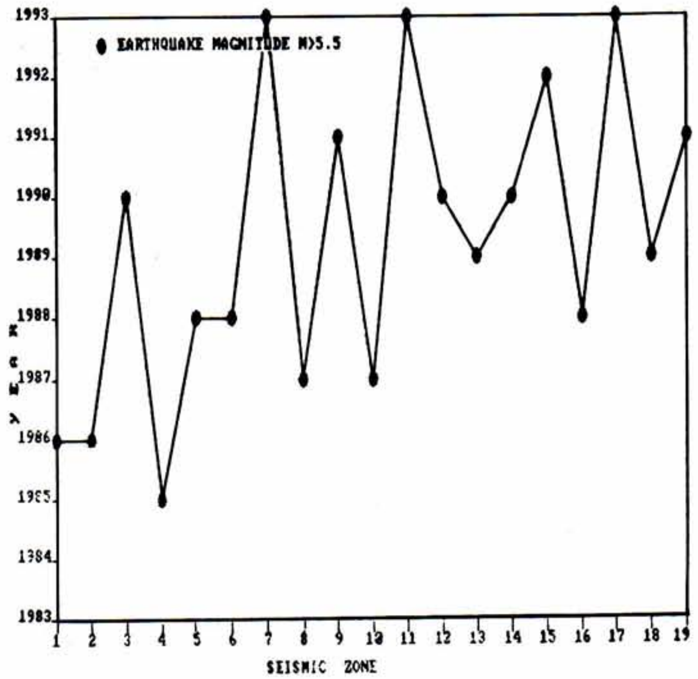

Fig. 7 — Expected year of earthquake occurrence with $M_j > 5.5$ for each seismogenic zone in Greece.

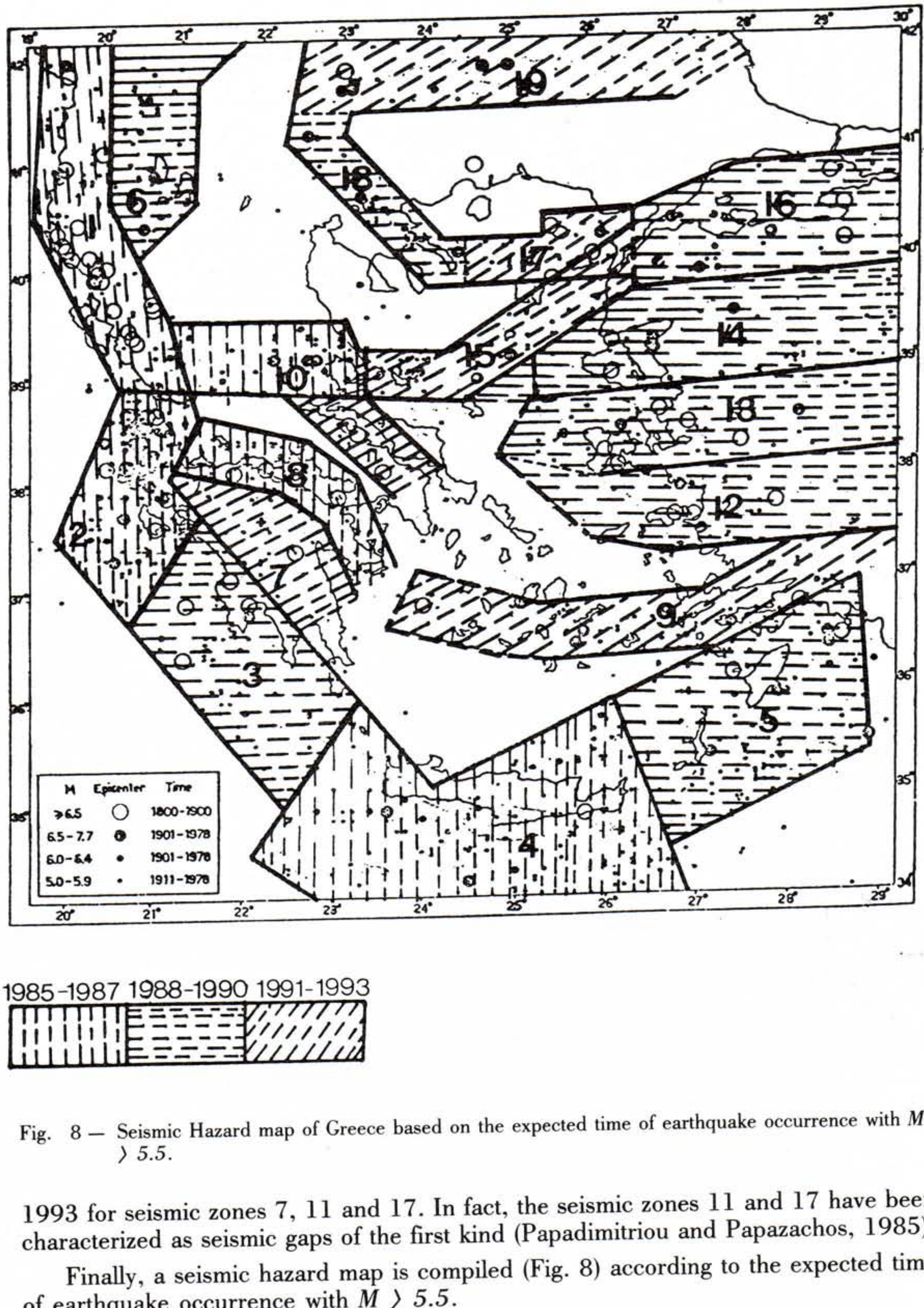

Fig. 8 — Seismic Hazard map of Greece based on the expected time of earthquake occurrence with $M_i \rangle 5.5$.

1993 for seismic zones 7, 11 and 17. In fact, the seismic zones 11 and 17 have been characterized as seismic gaps of the first kind (Papadimitriou and Papazachos, 1985).

Finally, a seismic hazard map is compiled (Fig. 8) according to the expected time of earthquake occurrence with $M \rangle 5.5$.

It should be emphasized that our results, obtained by advanced statistics, can be considered as a small contribution to the complicated problem of the long term earthquake prediction.

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