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AN INVESTIGATION OF THE PRINCIPLES OF OPERATION OF THE HEAT-PULSE FLOWMETER

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ABSTRACT

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The main limitation of velocity-measuring devices that have been available for use down boreholes is that they operate imperfectly or not at all below velocities of the order 10—30 mm s^{-1}. A new type of instrument named heat-pulse flowmeter has been used recently for velocity measurements as low as 5 mm s^{-1} with much success. The purpose of this paper is to investigate the principles of operation of a similar device and discuss the results of some experimental investigations.

INTRODUCTION

Fluid velocity logs are essential for the study and analysis of in situ flow, and hence for the assessment of in situ hydraulic conductivity. Obviously these logs rely on tools that must be capable of operating over a wide range of flows and the accurate measurement of low flows can be as valuable to analysis as that of high flows.

The basic idea of the heat-pulse flowmeter is to use the thermal wake of a heated wire, submerged into the moving fluid, as a tracer. The technique described here was first reported by Kowasznay (1949) and some years later by Walker and Westenberg (1956), but in each case was applied in air flow. All of these authors were interested in producing a periodical temperature field by applying a sinusoidal current in the probe, and trying to correlate the fluid velocity with phase measurements of the thermal wake.

The first worker who applied a single heat pulse into the fluid (air) was Bauer (1965). More recently, Tombach (1969) in the U.S.A. and Bradbury and Castro (1971) in England have been using modified versions of Bauer's original probe in highly turbulent flows.

All the above workers have used the instrument in a velocity range where the probe works essentially by convection with thermal diffusion having

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secondary influence. The analysis presented here is the opposite of previous efforts in that with low flows of the type that can be found in boreholes there is essentially a thermal diffusion problem being influenced by convection. If the influence of diffusion, as in the case of Bauer and Bradbury—Castro, was secondary, the time of travel for the heated cloud of fluid would correspond to the time between the pulse and the initial response of the downstream sensor. The response would then rise with a linear slope determined by the thermal inertia of the sensor wire, and with thermal diffusion tending to diminish this response (Blackwell, 1954; Parsons and Mulligan, 1978).

Fig. 1 represents the response of the sensor with conditions of differing thermal diffusion and sensor wire inertia. Analysis of the convection—diffusion problem was performed by both Tombach (1969) and Bradbury and Castro (1971) in order to find when the effects of diffusion reached the sensor. They would then be able to extrapolate back to zero response and find the precise travel time. The technique used by Tombach was to digitize the signals and analyse them on a computer. Bradbury and Castro used a convenient voltage offset to indicate the travel time.

Unfortunately in our case, because of the large effect of diffusion, the initial response of the sensor is going to be insensitive to convection at the low velocities that are of interest in borehole logging. This is the reason why it is preferable to use the location of the maximum signal for measuring travel time, since it is quite sensitive to the fluid velocity (Dudgeon et al., 1975). It is obvious that in order to predict the time of occurrence of the maximum signal in the sensor we have to predict the whole shape of the thermal cloud produced by the pulsed wire, the effect of diffusion on this cloud, the convection pattern of the viscous wake and, of course, the response of the sensor. This problem is very difficult to analyse theoretically, and in order to simplify it, the thermal inertia of the sensor and the pulsed

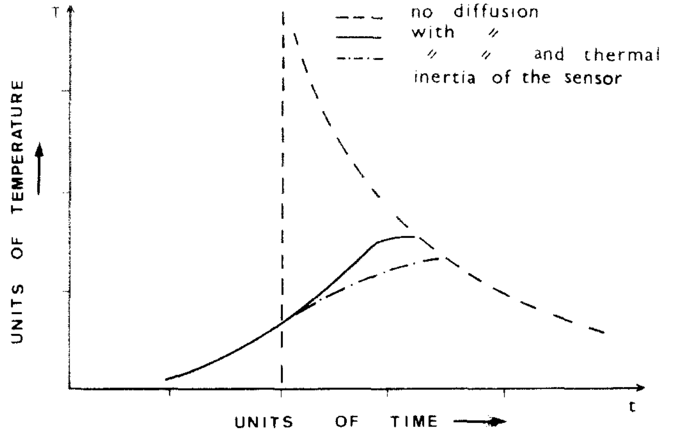

Fig. 1. Typical responses of a temperature sensor with conditions of different diffusion and sensor thermal inertia.

wire as well as the conduction of heat to the wire supports have been considered as less important. On the other hand, the most important aspects of the problem are determining the initial shape of the thermal cloud and assessing the convection—diffusion interaction.

FORMULATION OF THE PROBLEM

Water in laminar flow with initial temperature T_a passes a small wire of length l and diameter d. If the wire is heated by an electric pulse of known characteristics, we want to know the temperature field at a point A which is a distance L downstream (Fig. 2). See the Notation for symbols used in this paper.

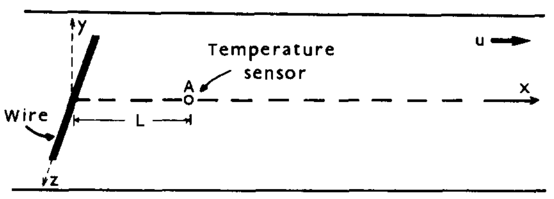

Fig. 2. The temperature field to be described at point A.

The differential equation of conduction of heat in an isotropic medium, whose thermal conductivity is independent of the temperature, is given by:

$$\partial^2 T/\partial x^2 + \partial^2 T/\partial y^2 + \partial^2 T/\partial z^2 - \hat{k}^{-1}(\partial T/\partial t) = 0 \quad (1)$$

On the other hand, the total amount of heat released into the medium is given (Carslaw and Jaeger, 1959):

$$\int\!\!\int\!\!\int_{-\infty}^{\infty} pcT \mathrm{d}x\mathrm{d}y\mathrm{d}z = Qpc/(8\pi \hat{k}t)^{3/2} \int_{-\infty}^{\infty} \exp[-(x-x')^2/4\hat{k}t]\,\mathrm{d}x \times \int_{-\infty}^{\infty} \exp[-(y-y')^2/4\hat{k}t]\,\mathrm{d}y \int_{-\infty}^{\infty} \exp[-(z-z')^2/4\hat{k}t]\,\mathrm{d}z = Qpc \quad (2)$$

From this, the solution of eq. 1 may be interpreted as the temperature due to a quantity of heat Qpc released instantaneously at $t = 0$ at the point (x',y',z'). Transferring eq. 1 to a coordinate system fixed with respect to the source:

$$\partial^2 T/\partial x^2 + \partial^2 T/\partial y^2 + \partial^2 T/\partial z^2 - \hat{k}^{-1}[(\partial T/\partial t) + u(\partial T/\partial x)] = 0 \quad (3)$$

If the heat emitted at the origin by the point-source is described by the

NOTATION

List of symbols

a_0	first temperature coefficient of resistance
β	volume coefficient of expansion of water
c	specific heat
d	diameter
g	acceleration of gravity
Gr	Grashof's number
k	thermal conductivity
$\hat{k}$	thermal diffusivity
l	length of the wire
L	distance between temperature sensor and pulsed grid
m	number of wires in the grid
Nu	Nusselt's number
P	Peclet's number
Pr	Prandtl's number
Q	heat
R	resistance
Re	Reynolds' number
p	density
σ_0	resistivity
T	temperature
T_1	initial temperature of the grid at the end of the heating pulse
τ_p	time constant for appreciable convection
t	time
u	velocity
V	voltage
v	kinematic viscosity
x, y, z	Cartesian coordinates
μ	dynamic viscosity

In all the above symbols index w means wire; f means fluid; and a means ambient.

function $q(t)$, then the temperature at time t at point (x,y,z) due to the quantity of heat $q(t')\mathrm{d}t'$ released at time t' is:

$$[q(t')\mathrm{d}t']/[8pc\{\pi\hat{k}(t-t')\}^{3/2}] \times \exp\left(-[\{x-u(t-t')\}^2+y^2+z^2]/[4\hat{k}(t-t')]\right) \quad (4)$$

and the temperature at time t due to the heat emitted at the origin from time 0 to time t is:

$$T(x,y,z,t) = 1/[8pc(\pi\hat{k})^{3/2}]\int_0^t [q(t')]/[(t-t')^{3/2}] \times \exp\left(-[\{x-u(t-t')\}^2+y^2+z^2]/[4\hat{k}(t-t')]\right)\mathrm{d}t' \quad (5)$$

For a large aspect ratio, instead of a point source we have an infinite line-source along the y-axis, and eq. 5 becomes:

$$T(x,y,t) = 1/(4\pi\hat{k}pc)\int_0^t q(t')\cdot\exp[-r^2/\{4\hat{k}(t-t')\}]\,dt'/(t-t') \qquad (6)$$

where

$$r^2 = [x-u(t-t')]^2 + y^2 \qquad (7)$$

From eq. 6 it becomes clear that in order to determine the temperature distribution downstream, we have to know the functional form of $q(t)$, i.e. the physical law which describes the heat transfer from the pulsed wire. This can be easily calculated as long as the temperature history of the pulsed wire can be found. Thus $q(t)$ can be written as (Collis and Williams, 1959):

$$q(t) = \pi k\cdot \mathrm{Nu}\cdot(T(t)-T_a)\cdot l \qquad (8)$$

where Nu is the dimensionless Nusselt number. Recent measurements have shown that at low Reynolds numbers, the Nusselt number depends on the Reynolds number, Re, the Prandtl number, Pr, and the angle of inclination of the fluid stream to the wire, neglecting the buoyance effect. The determination of the precise functional form of Nu (Re, Pr) is a classical problem in fluid mechanics, and although it is of great importance for the study of hot-wire anemometers it is not so important in the present analysis, where the modified Kramers equation (Castaldini et al., 1980) has been adopted as representing the heat convection process:

$$\mathrm{Nu} = 0.42\cdot \mathrm{Pr}^{0.2} + 0.57\cdot \mathrm{Pr}^{0.33}\cdot \mathrm{Re}^{0.36} \qquad (9)$$

From the principle of conservation of energy the following equation can be written:

(electric energy entering the pulsed wire) = (heat stored in the wire)
+ (heat convected by the water)

or

$$V^2/R = p_w c_w \pi(d^2 l)/4 + \pi k l \mathrm{Nu}(T-T_a) \qquad (10)$$

From which the time constant for appreciable convection becomes:

$$\tau_p = p_w c_w d^2/4k\mathrm{Nu} \qquad (11)$$

By keeping the duration of the electric pulse well below τ_p the heat convection term in eq. 10 can be ignored, and the equation reduces to the form:

$$(a_0/2)(T-T_a)^2 + (T-T_a) = (V/l)^2 t/(pc\sigma_0) \qquad (12)$$

the solution of which gives the temperature history of the wire during the pulsing time.

After the nearly instantaneous rise of temperature, the pulsed wire temperature decays exponentially due to heat convection, according to the equation:

$$(T-T_a) = (T_1-T_a)\cdot\exp(-t/\tau_p) \qquad (13)$$

where T_1 is the initial temperature of the probe at the end of the pulse. When the water velocity is low the time for the temperature decay in the pulsed wire will be much shorter than the time of diffusion between the pulsed wire and the temperature sensor, a fact which permits the assumption of an instantaneous source at the velocity range of interest. Under these conditions, the temperature field at point A (Fig. 2), was found for the

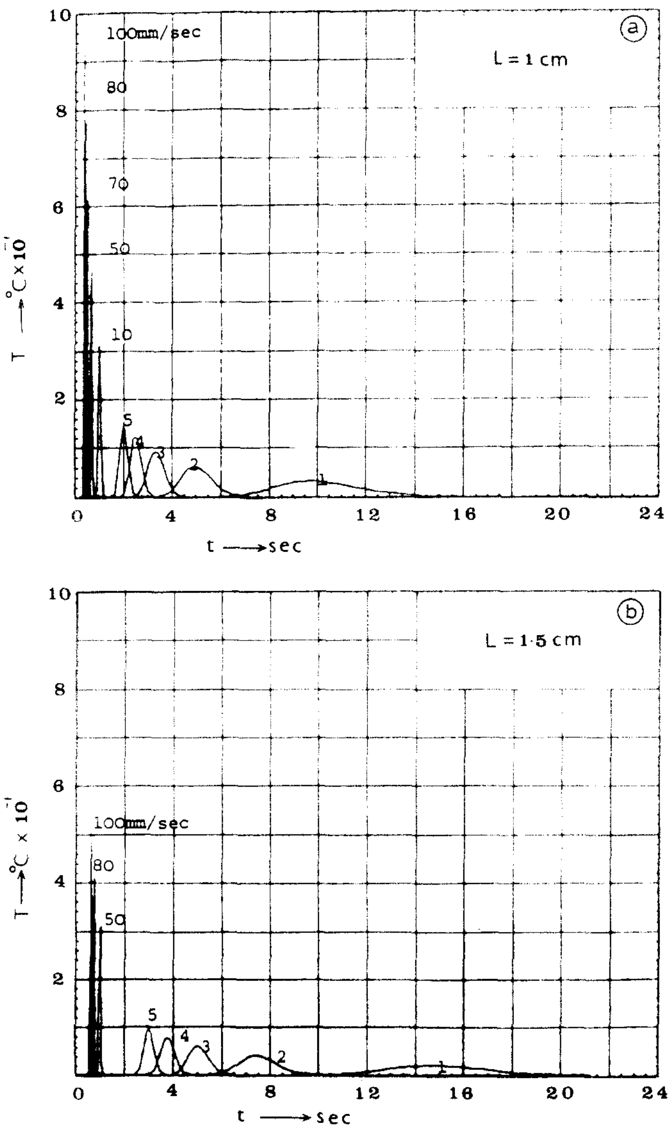

Fig. 3. Temperature at point A as a function of time and water velocity for: (a) $L = 1$ cm; and (b) $L = 1.5$ cm.

case of a pulsing grid consisting of m wires 1 mm spaced, to be given by the following expression:

$$(T - T_a) = m[(T_1 - T_a) \cdot \mathrm{Nu} \cdot \tau_p]/4t \cdot \exp[-\{\delta \cdot (1 - tu/L)^2\}/(tu/L)] \times [1 + \{\tau_p/(t^2 \cdot u/L)\} \cdot t^2 \cdot \delta \cdot (u/L)^2 + t \cdot u/L - \delta] + \mathrm{GF} \quad (14)$$

where GF is a negative factor depending upon the geometry of the pulsed grid (see the Appendix).

Fig. 3a and b represents the temperature field at A, as a function of time and the velocity of the water, for 1- and 1.5-cm grids to sensor separation respectively as it was calculated from eq. 14.

DESIGN CRITERIA AND CALIBRATION OF A HEAT-PULSE FLOWMETER

The design of a heat-pulse flowmeter, and the way that it operates, depend not only on the dynamics of the convection and diffusion of the heat pulse but also on some of the practical electronic and mechanical constraints that arise. These are problems of signal-to-noise ratio and the influence of electrical coupling between the pulsed wire and the sensor circuit.

The probe design and its electrical circuitry

It is evident that the temperature decay of the pulsed wire is faster when its diameter is decreasing. This produces a more sharply peaked thermal cloud moving between the pulsed grid and the sensor. Therefore it is easier to determine the position of maximum signal in the sensor. Unfortunately, because there is an upper limit to the temperature to which we can heat the pulsed wire, its dimensions cannot be reduced a lot if enough energy is to be transferred to the water.

The most important parameter is the distance between the pulsed wire and the temperature sensor, because both the convection time and the Peclet number depend on it. From heat conservation arguments it is expected that the amplitude of the temperature sensor's signal is not strongly affected by the changing of spacing between the sensor and the heating grid, but in the present case the effect of longitudinal diffusion is so strong that the signal peak decays very rapidly with the spacing. On the other hand, noise will be directly proportional to the total wire length of the grid; this includes both electronic noise from the amplifier and also any response that the sensor has directly to the water stream flowing past it, either to temperature fluctuations as a resistance thermometer or to velocity fluctuations as a hot-wire flowmeter. It would appear, therefore, that there is a strong motive to reduce the spacing between the wire and the sensor as much as possible, keeping in mind the effect that the grid's viscous wake has on the sensor.

Another difficult task is to choose the appropriate temperature sensor. The decision must be based on the following objectives:

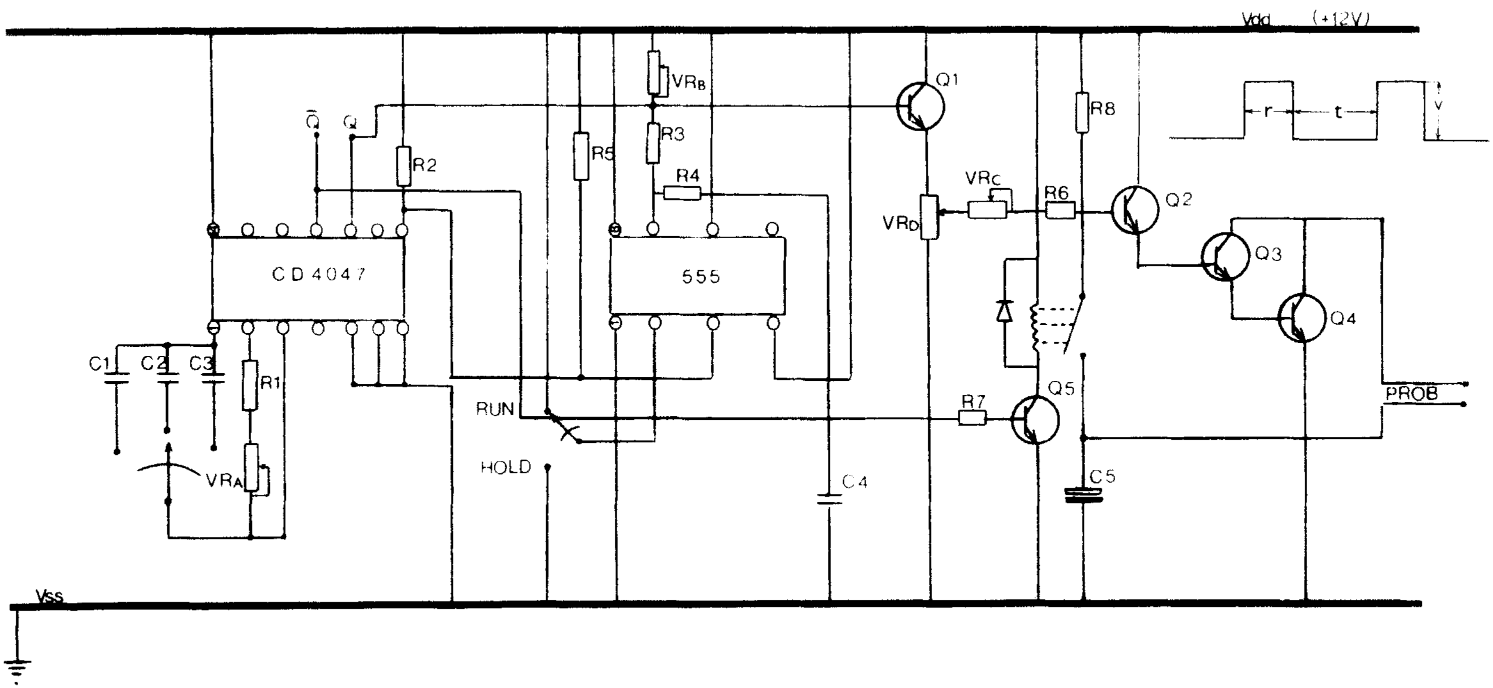

Fig. 4. Electronic circuit of the pulse generator. Time *r* is controlled by VRA, time *t* is controlled by VRB and the amplitude of the pulse can be controlled by potentiometers VRC and VRD.
Components: R*1* = 10K; R*2* = 22K; R*3* = 1M; R*4* = 100K; R*5* = 10K; R*6* = 27K; R*7* = 100K; R*8* = 1K.
C*1* = 0.022 F; C*2* = 0.22 F; C*3* = 2.2 F; C*4* = 2.2 F; C*5* = 2000 F.
Q*1* = BC*107*; Q*2* = BC*107*; Q*3* = 2N*3054*; Q*4* = 2N*3055*; Q*5* = BC*107*.

(1) Provide, as far as possible, a direct means of heat-wave measurement without the need of complex corrections.

(2) Obtain a sufficiently good response characteristic.

(3) Provide a sensing element which is reliable in a water environment, up to temperatures of ~100°C, without drift problems with time, and giving a long working life.

The ideal sensor would be a fine wire, but because of contamination due to dirt problems (Tselentis, 1982) a second choice could be a thermistor capable of measuring at least 0.01°C and having a response time not greater than 0.1 s.

In the present experiment, the duration of the pulses used to produce the tracer of heated water ranges from a few microseconds up to ~100 ms with peak currents as high as 3 A. There is nothing intrinsically difficult about making a circuit to produce such pulses, and Fig. 4 shows the circuit designed for the present experiment. Generally in the design of the electric circuit of a heat-pulse flowmeter, capacitor discharge must be employed for pulsing the heating element. This technique permits the high-power pulses required to be generated down the hole with minimal difficulty and results in a smooth and continuous supply of power flowing in the cable link. This is a desired feature since cross-coupling, in a system where the input peak power is considerably greater than the signal power, invariably results in sacrificed sensitivity, and a high degree of isolation is required between the two circuits (pulsed wire and sensor) in the probe.

Calibration

The most common method used for the calibration of standard flowmeters is to place the probe in a flow and measure the velocity by a second "standard" method, e.g., with a Pitot static probe. Unfortunately these standard methods become inaccurate at very low velocities. A second method consists of holding the fluid stationary and moving the probe through it (Newman, 1966). The major shortcoming of this method is the resultant mechanical vibration of the probe. A similar method is to hold the probe stationary and move the container with the fluid (Dring and Gebhart, 1968). Various other arrangements have been used, such as towing, towed and rotating tanks, etc. In the present experiment, an adjustable reservoir tank has been used, supplying water through a flexible hose to the bottom of a vertical calibration tube. The reservoir can be moved very accurately by a mechanism so that the difference in head can be easily changed, allowing for the water to overflow in both the reservoir and the calibration tube in order to achieve constant head. By having a wire screen (15—20 mesh/in.) below the probe and leaving an opening of ~2 mm between the rim of the screen and the tube wall, a more flattened profile near the centre line (Callagher, 1973) can be obtained. This makes exact probe placement less critical than in the case of unobstructed flow. On the other hand, the velocity at the tube axis can be made very close to the cross-tube average by adjusting the gap between the screen and the tube walls. So the tube output can be related directly to the measured mean-flow speed and computations involving the velocity profile can be eliminated. The system itself is calibrated by adjusting the reservoir surface at various elevations and measuring the volume outflow or weight during a known time.

DISCUSSION OF THE RESULTS

Measurements of the travel time to the maximum amplitude of the temperature sensor's signal have been obtained from chart-recorder traces. These

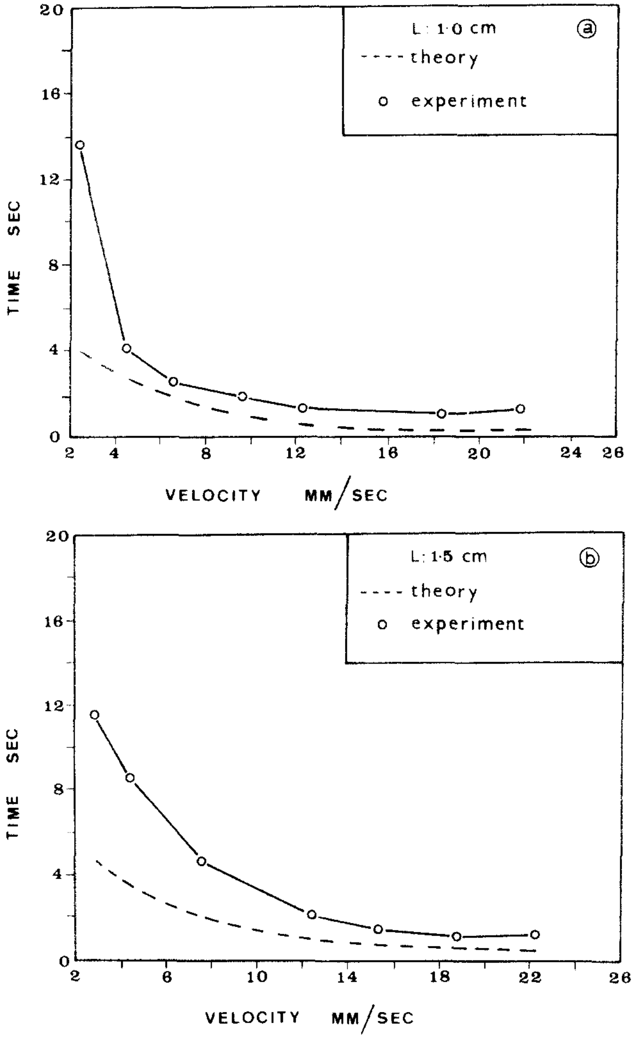

Fig. 5. Comparison between theoretical and experimental results of a 0.25 mm Pt wire for: (a) $L = 1$ cm; and (b) $L = 1.5$ cm.

measurements can conveniently be used to give an indication of the agreement between the theoretical and the experimental results and are presented in Fig. 5a and b for a heat source to sensor separations of 1 and 1.5 cm, respectively, for a 0.25-mm-diameter Pt wire. The agreement is quite satisfactory in showing the general trends.

One fundamental limitation of the theoretical analysis for low velocities is the assumption that the thermal diffusivity of water is constant, in order to solve eq. 1. It is known that the thermal diffusivity of water varies significantly with temperature, so that as the thermal pulse diffuses, it cools, decreasing the value of $\hat{k}$.

The comparison between the theoretical and experimental results shows a significant increase in difference as the velocities decrease (Fig. 6). One obvious reason for this is that the time of travel increases, giving the thermal pulse more time to diffuse and cool. The previous discussion of heat transfer from the grid to the water considered only the calculation of forced convection where water is forced through the heat-transfer grid. In the present case, because of the very low velocities encountered, the forced-flow velocity is coupled with the convective velocity which is generated by the buoyancy forces resulting from a reduction in fluid density near the heated grid. As the water velocity increases, the forced convective velocity would be expected to overshadow most free-convection effects encountered in the system. On the other hand, flow situations of very low water velocities might be appreciably influenced by free convection currents.

An order of magnitude analysis of the free-convection boundary layer equation indicates that the natural convection is significant when (Holman, 1976):

$$\mathrm{Re} < \mathrm{Gr}^{1/3} \tag{15}$$

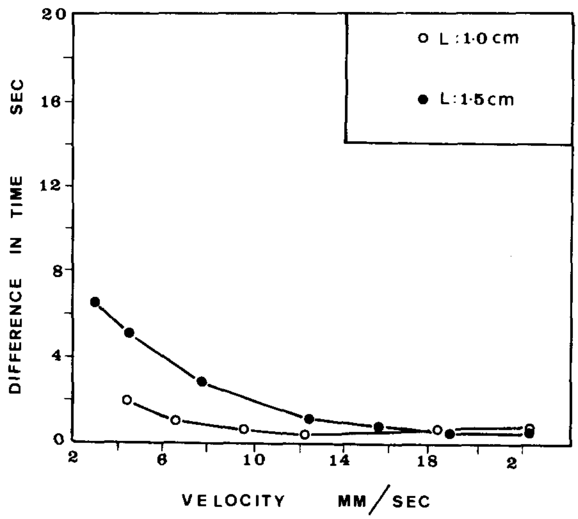

Fig. 6. Difference between theoretical and experimental results as a function of the fluid velocity.

where Gr is a dimensionless number (Grashof's number) representing the ratio of a typical buoyancy force to a typical viscous force and is given by the following relation:

$$\mathrm{Gr} = (g \cdot \beta \cdot d^3/v^2) \cdot (T - T_a) \quad (16)$$

Calculations showed that for velocities of less than 4 mm s^{-1} a mixed convection situation is to be expected.

CONCLUDING REMARKS

This paper has described the results of both theoretical and experimental investigations into a pulse-grid probe using a time of travel technique for water velocity measurements.

However, although the present work represents a start on the theoretical and practical analysis of the method, it is clear that there are many aspects of the device's behaviour that require further investigation. There are, of course, other techniques developed for use in water wells, but the main limitation of most of them is the relatively high threshold velocity below which they will not operate. With the heat-pulse flowmeter we can measure velocities as low as 4 mm s^{-1}, something which permits us to study the flow regime in water wells, even under natural gradient conditions. Having no moving parts, the instrument operates over a wide dynamic range in a wide variety of conditions.

APPENDIX

Derivation of expression (14)

Expressing eq. 3 in terms of dimensionless variables:

$$\partial^2 \hat{T}/\partial \xi^2 + \partial^2 \hat{T}/\partial \eta^2 = P \cdot (\partial \hat{T}/\partial \tau + \partial \hat{T}/\partial \xi) \quad (\text{A-1})$$

where

$$\left.\begin{aligned} \hat{T} &= (T - T_a)/T_a \\ \xi &= x/L; \qquad n = y/L; \qquad r = tu/L \\ P &= uL/\hat{k} \qquad \text{(Peclet's number)} \end{aligned}\right| \quad (\text{A-2})$$

The solution of eq. A-1 can be written as:

$$T(\xi,n,r) = P/4pc \int_0^{\tau} q(t') \cdot \exp[-P\{(\xi - \tau')^2 + \eta^2\}/4\tau'] \, d\tau'/\tau' \quad (\text{A-3})$$

and because of eqs. 13 and 8, eq. A-3 can be written as:

$$T(\xi,n,r) = \mathrm{Nu}\cdot T_1/4\int_0^{\tau} \exp[-(\tau-\tau')/\tau_p] \times \exp[-P\{(\xi-\tau')^2+\eta^2\}/4\tau']\,d\tau'/\tau' \qquad \text{(A-4)}$$

Considering the plane which passes through A and contains the wire, the dimensionless variable n takes the value 0 and to study the temperature field at the point A, ξ takes the value 1. For this case, eq. A-4 can be written as:

$$T(r) = \mathrm{Nu}\cdot T_1/4\int_0^{\tau} \exp[-(\tau-\tau')/\tau_p]\cdot\exp[-P\cdot(1-\tau')^2/4\tau']\,d\tau'/\tau' \qquad \text{(A-5)}$$

Because the velocities being studied with the heat-pulse flowmeter, when it is used for groundwater measurements, are relatively low, the dimensionless decay time of the pulsed wire is very small compared to the dimensionless travel time between the wire and the sensor, so the assumption of an instantaneous pulse is made. It is well known from the theory of conduction of heat that the solution of eq. 3 for the case of an instantaneous heat source has the form:

$$T(x,y,t) = (Q/4\pi\hat{k}t)\cdot\exp(-r^2/4\hat{k}t) \qquad \text{(A-6)}$$

In dimensionless form, eq. A-6 can be written as:

$$T(\xi,r) = (QP/4\pi r)\cdot\exp[\{-(\xi-r)^2\}\cdot P/4r] \qquad \text{(A-7)}$$

from which the time to the maximum signal becomes $P/4$. It is convenient, therefore, to rescale the dimensionless time r in eq. A-4 by putting:

$\tau' = \delta\cdot\lambda$ and $r = \delta\cdot\mu$ where $\delta = P/4$

so we have:

$$T(r) = \mathrm{Nu}\cdot T_1/4\int_0^{\mu} \exp[-\delta\cdot(\mu-\lambda)/\tau_p]\cdot\exp[-(1-\delta\lambda)^2/\lambda]\,d\lambda/\lambda \qquad \text{(A-8)}$$

and because $r_p/\delta \ll 1$ the above integral is very small for all the values of λ except for $\lambda = \mu$, so by defining a new variable of integration:

$$\lambda = \mu - \tau_p\cdot\lambda'/\delta \qquad \text{(A-9)}$$

the following integral equation is obtained:

$$T(r) = (\mathrm{Nu}\cdot T_1\cdot\tau_p)/4\delta \int_0^{\mu\delta/\tau_p} \exp[-\lambda'-(1-\delta\mu+\tau_p\cdot\lambda')/(\mu-\tau_p\cdot\lambda'/\delta)] \times[(d\lambda')/(\mu-\tau_p\cdot\lambda'/\delta)] \qquad \text{(A-10)}$$

By expanding the factor $1/[\mu(1-\tau_p\cdot\lambda'/\delta\mu)]$ in terms of $\tau_p\cdot\lambda'/\delta\mu$:

$$T(\tau) = (\mathrm{Nu}\cdot T_1\cdot\tau_p)/4\delta\cdot\int_0^{\mu\delta/\tau_p} \exp(-\lambda')\cdot\exp[-\{(1-\delta\mu)^2\}/\mu - (\tau_p/\delta)\{(1-\delta^2\mu^2)^2\}/\mu^2\cdot\lambda' - (\tau_p/\delta)^2\cdot(\lambda'^2/\mu^3) + \mathrm{O}(\tau_p/\delta)^3] \times 1/\mu\cdot[1+(\tau_p/\delta)\cdot(\lambda'/\mu)+(\tau_p\cdot\lambda'/\delta\mu)^2+\ldots]\,\mathrm{d}\lambda' \qquad (\text{A-11})$$

Eq. A-11 can be expanded in terms of $\exp(\tau_p/\delta)$ and $\exp(\tau_p/\delta)^2$, when it becomes a simple exponential integral which gives:

$$\hat{T}(\tau) = (\mathrm{Nu}\cdot\hat{T}_1\cdot\tau_p/4\delta\mu)\exp[-(1-\delta\mu)^2/\mu]\Big(1+(\tau_p/\delta)\times\{(\delta^2\mu^2+\mu-1)/\mu^2\}+(\tau_p/\delta)^2\cdot[\{(\delta^2\mu^2+\mu-1)/\mu^2\}^2 + 1-(2/\mu)+\mathrm{O}\{(\tau_p/\delta)^3,\exp(-\delta\mu/\tau_p)\}]\Big) \qquad (\text{A-12})$$

and by transferring eq. A-12 to the initial coordinate system it gives eq. 14 for $n=1$ and GF$=0$.

The temperature field produced by a grid

In the following, an assumption is made of a grid consisting of m wires each of length 1 cm and 1 mm apart. Obviously, the dimensionless variable n takes the following values:

$$n = 1/L\cdot(0.5+k) \qquad (\text{A-13})$$

where $k=0, 1, 2, 3, \ldots, m-1$, so eq. A-4 can be written:

$$T(\eta,\tau) = (\mathrm{Nu}\cdot T_1/2)\sum_{k=0}^{m-1}\int_0^{\tau}\exp-(\tau-\tau')/\tau_p \times\exp-P\cdot[(1-\tau')^2+\{(0.5+k)/L\}^2]\,4\tau'\,\mathrm{d}\tau'/\tau' \qquad (\text{A-14})$$

and, using the same analytical methods as above, eq. A-14 can be written:

$$T(\eta,\tau) = (m\cdot\mathrm{Nu}\cdot T_1\cdot\tau_p)/4\delta\mu\cdot\exp[-(1-\delta\mu)^2/\mu]\cdot[1+(\tau_p/\delta)\cdot\{(1/\mu) - (1-\delta^2\mu^2)/(\mu^2)\}+2(\tau_p/\delta)^2\cdot\{(1-\delta^2\mu^2)^2/(2\mu^4)-(1/\mu^3) + (1/\mu^2)-(1-\delta^2\mu^2)/(\mu^3)\}]+(\mathrm{Nu}\cdot T_1\cdot\tau_p)/(2\delta\mu) \times\exp[-(1-\delta\mu)^2/\mu]\cdot\sum_{k=0}^{m-1}\exp[-\eta(k)]\cdot[2(\tau_p/\delta)^2(1-\delta^2\mu^2)^2/(\mu^4) - \{(\tau_p/\delta)\cdot(1/\mu^2)\}+(\tau_p/\delta)^2\cdot\{\eta(k)\}^2/\{\mu^4\}] \qquad (\text{A-15})$$

By comparing eq. A-15 with eq. A-12, it becomes obvious that when using a grid consisting of m wires the temperature field at point A is m times the temperature field caused by a single wire, plus a negative "grid factor", GF. Calculations for GF for the case of a grid consisting of 10 wires gave very small values (10^{-6}), so GF can be neglected without introducing any significant error in the calculation of the temperature field.

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