Geophysics, Gravity, Microcomputers

A SIMPLE METHOD FOR GRAVITY BASE NETWORK ADJUSTMENT BY THE USE OF A MICROCOMPUTER

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A different approach in adjusting gravity base networks is presented by the use of microcomputers. The method used for solving the corresponding system of linear equations is the "Generalized Inverse", and a program written in Microsoft Basic for a PET - COMMODORE microcomputer is presented with an application example.

INTRODUCTION

Since the introduction and rapid growth of the microcomputer industry in the mid 1970's, the geophysical industry has been quick to make use of this technology, as microprocessing capability built into exploration geophysical equipment, and more recently, as small stand alone microcomputer systems can be used for interpretation in the office or in the field.

In the field, microcomputers are used to record and store survey data, on either cassete or floppy disk depending on the hardware used. Once entered, field data can be manipulated to give a hard copy output either as listing or as profiles.

Depending on the type of survey, the field computers are also used to carry out the necessary calculations and corrections, to reduce the raw field data to their final format. Examples of this being, reduction of gravity data, magnetic corrections, plotting of data e.t.c.

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As an interpretational aid, the field computer is used to model the geophysical results in terms of geologic models to provide quantitative values to the anomaly parameters. All the modelling and interpretation routines developed are interactive programs which allow the geophysicist and geologist maximum input to all phases of the computer manipulation. Thus constraints based on the geologic knowledge can be input to the computer aided geophysical interpretation.

This paper presents an example of the use of microcomputers in the topic of gravity survey problems and more specifically on the adjustment of gravity networks.

GRAVITY NETWORK ADJUSTMENT

A gravity network adjustment consists of correcting each gravity difference measured between two adjacent bases so that the closing error along a closed loop is minimum.

That problem was faced by Barton (1929) and Roman (1932), using the method of least - squares. Another approach was followed by Cowles (1938) in a similar problem, where he proposed a method of writting down a system of simultaneous equations for correction of closing slope data traverses. That system of equations can be solved by means of an analogous resistance network. An itterative graphical method for solving the corresponding system of equations was proposed by Gibson (1941). Smith (1950) also proposed another graphical method based on Gibson's work.

The use of computer methods in solving the corresponding system of simultaneous equations simplified the problem of adjusting gravity networks. In that field various computer programs in FORTRAN 4 exist in the geophysical computer libraries. Recently, Lagios (1984) presented an analysis of a least - squares adjustment of a gravity base station network and a computer FORTRAN 4 program in which the instrumental drift has been considered.

A review of the methods used of the adjustment of geodetic networks was given by Welsch (1979). The same methods can be applied generally on gravity base networks.

FUNDAMENTAL THEORY

Let us consider the gravity base network of figure (1). The values at the centre of each triangle represent the closing error of each loop of the network. The letters a, b, c, ... m, represent the corrections to be applied at each traverse between two bases, so that the closing error of each closed loop (triangle) is zero (or minimum).

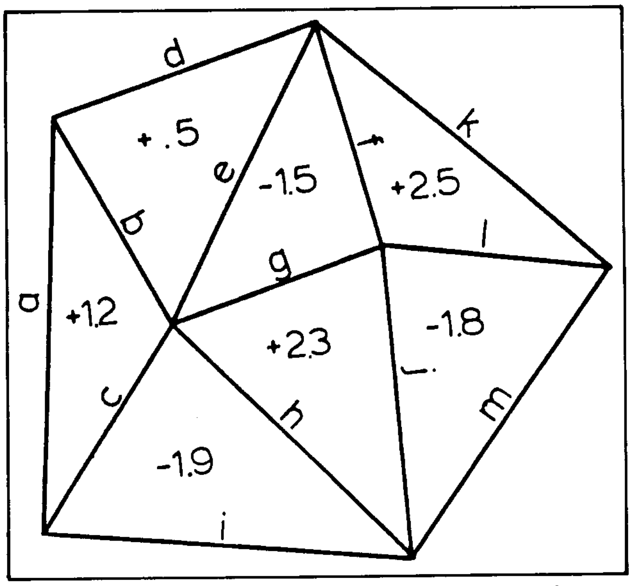

FIGURE 1: *Gravity base station network.*

From each triangle an equation results of the form:

$$a + b + c + ce = 0 \qquad (1)$$

where a, b, c are the corrections to be applied on each traverse respectively for each triangle, and ce corresponds to the closing error of the same triangle. Signes are considered to be positive clockwise.

For the gravity network of figure (1) the following system of equations (fig. 2) holds, presented in a matrix form.

$$\begin{Vmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & -1 & 1 \end{Vmatrix} \cdot \begin{vmatrix} a \\ b \\ \vdots \\ l \\ m \end{vmatrix} = \begin{vmatrix} -1.2 \\ -0.5 \\ 1.5 \\ -2.5 \\ -2.3 \\ 1.9 \\ 1.8 \end{vmatrix}$$

FIGURE 2: *Resulted system of linear equations.*

Since the number of unknowns is greater than the resulted independed linear equations that system can't be solved by the classical methods. In order to override that difficulty the generalized inverse method is used (Mackay, 1981). In that method it is possible by using an itterative algorithm to calculate the generalized inverse $A_{+}(N,M)$ of an $A(N,M)$ matrix (Moore - Penrose Inverse), so that the matrix equation:

$$A(N,M)\cdot X(N,1) = H(N,1) \qquad (2)$$

can be solved as

$$X(N,1) = A_{+}(N,M)\cdot H(M,1) \qquad (3)$$

This procedure can solve the following three kind of problems:

a) If N=M and the A(N,M) exists, then we get an exact solution for X(N,1).

b) If M>N that is the equations are more than the number of unknowns, then a least - squares solution is given instead.

c) If N>M that is the equations are fewer than the number of unknowns, then a solution is given consistent with the data (minimum norm g^{+} inverse).

The following results, presented in figure (3) have been obtained by the method reffered above using Mackay's algorithm on the system of equations of figure (2).

a	=	-.4333333330
b	=	-.0772357722
c	=	-.6894308940
d	=	-.3560975610
e	=	-.2211382110
f	=	.7154471520
g	=	.5634146310
h	=	.9544715420
i	=	.2560975610
j	=	-.7821138180
k	=	-.8504065020
l	=	-.9341463400
m	=	.0837398381

FIGURE 3: *Derived solution of linear system.*

PROGRAM USER INSTRUCTIONS

The computer program written originally in Microsoft Basic was slightly modified to meet the output requirements of the PET - COMMODORE computer operating system.

The user of the program should follow these instructions:

1) Enter number of equations and number of unknowns in data statement 1440.
2) Enter the coeficients of each linear equation in successive data statements, one statement for each equation.
3) Run the program.

```
2 REM          ****************
4 REM          *              *
6 REM          *   BASENET    *
8 REM          *              *
10 REM         ****************
12 REM
14 REM  THIS PROGRAM ADJUSTS THE MISCLOSURES OBSERVED ON A GRAVITY NETWORK
16 REM
18 REM  ON EACH SIDE OF THE TRIANGLES A CORRECTION A,B,C, ETC IS ASSIGNED AND
19 REM
20 REM  THE CORRESPONDING LINEAR EQUATIONS ARE FORMED.
21 REM
22 REM  SINCE THE NUMBER OF LINEAR EQUATIONS IS LESS THAN THE NUMBER OF UNKNOW
23 REM
24 REM  NS, THE LINEAR SYSTEM IS SOLVED WITH THE GENERALIZED INVERSE METHOD
26 REM
28 REM  INPUT DATA CONSISTS OF:
30 REM
32 REM  1) NUMBER OF EQUATIONS
34 REM
36 REM  2) NUMBER OF UNKNOWNS
38 REM
40 REM  3) MATRICES OF LINEAR EQUATIONS IN THE FORM
42 REM
44 REM   X11 X12 X13 X14 Y1
46 REM
48 REM   X21 X22 X23 X24 Y2
50 REM
52 REM   X31 X32 X33 X34 Y3
54 REM
56 REM   ETC
58 REM
60 REM  EACH ROW CORRESPONDING TO A LINEAR EQUATION
62 REM
64 REM  DATA ARE ENTERED IN DATA STATEMENTS FROM 1440 TO 1510
66 REM
68 REM
70 REM
72 REM
74 REM
100 OPEN1,4
110 CMD1
115 REM
120 REM ***    GENERALIZED INVERSE OF X(N,M)    ***
125 REM
130 READ N,M
140 DIM H(N),P(M),Q(N)
150 DIM X(N,M),Y(M,N),W(M,N),Z(N,N),A(M,M)
155 REM
160 REM ***    MATRIX ENTERED IN X AND RETURNED IN Y    ***
165 REM
170 REM ***    READ IN MATRIX    ***
175 REM
180 FOR I=1 TO N
190 FOR J=1 TO M
200 READ X(I,J)
210 W(J,I)=X(I,J)
215 REM
220 REM ***    W IS TRANSPOSE OF X    ***
225 REM
230 NEXT J
235 REM
240 REM ***    R.H.S OF EQUATION    ***
245 REM
250 READ H(I)
260 NEXT I
270 K=0
280 FOR I=1 TO N
290 FOR J=1 TO N
300 Z(I,J)=0
310 FOR L=1 TO M
320 Z(I,J)=Z(I,J)+X(I,L)*W(L,J)
330 NEXT L
340 K=K+ABS(Z(I,J))
350 NEXT J
360 NEXT I
370 K=1/K
380 PRINT "K";K
385 REM
390 REM ***    SMALL CONSTANT    ***
395 REM
400 D=1E-5
410 PRINT"CONSTANT FOR INTEGRAL TRACE =";D
420 PRINT"TRACE+2*N"
430 FOR I=1 TO M
440 FOR J=1 TO N
445 REM
450 REM ***    FIRST APPROXIMATION TO INVERSE    ***
455 REM
460 Y(I,J)=K*W(I,J)
470 NEXT J
480 NEXT I
490 FOR I=1 TO N
500 FOR J=1 TO N
510 Z(I,J)=0
520 FOR L=1 TO M
530 Z(I,J)=Z(I,J)+X(I,L)*Y(L,J)
540 NEXT L
550 NEXT J
560 NEXT I
565 REM
```

```
570 REM ***    TRACE=T    ***
575 REM
580 T=0
590 FOR I=1 TO N
600 Z(I,I)=Z(I,I)-2
610 T=T+Z(I,I)
620 NEXT I
630 PRINT 2*N+T
640 FOR I=1 TO M
650 FOR J=1 TO N
660 W(I,J)=0
670 FOR L=1 TO N
680 W(I,J)=W(I,J)+Y(I,L)*Z(L,J)
690 NEXT L
700 NEXT J
710 NEXT I
720 FOR I=1 TO M
730 FOR J=1 TO N
740 Y(I,J)=-W(I,J)
750 NEXT J
760 NEXT I
770 IF ABS(T-INT(T)-1)<D THEN 800
780 IF ABS(T-INT(T))<D THEN 800
790 GOTO 490
795 REM
800 REM ***    REPEAT UNTIL T IS AN INTEGER    ***
805 REM
810 FOR I=1 TO M
820 FOR J=1 TO M
830 A(I,J)=0
840 FOR L=1 TO N
850 A(I,J)=A(I,J)-Y(I,L)*X(L,J)
860 NEXT L
870 NEXT J
880 NEXT I
890 PRINT"RANK OF MATRIX=";2*N+T
895 REM
900 REM ***    REMOVE STATEMENT 910 FOR FULL PRINTOUT    ***
905 REM
910 GOTO1250
920 PRINT "GENERALIZED INVERSE"
930 PRINT
940 FOR I=1 TO M
950 FOR J=1 TO N
960 PRINT Y(I,J);
970 NEXT J
980 PRINT:PRINT
990 NEXT I
995 REM
1000 REM ***    CHECKING PROCEDURE    ***
1005 REM
1010 PRINT
1020 PRINT"ORIGINAL MATRIX"
1030 FOR I=1 TO N
1040 FOR J=1 TO M
1050 PRINT X(I,J);
1060 NEXT J
1070 PRINT
1080 PRINT
1090 NEXT I
1100 PRINT
1110 PRINT"PRODUCTS"
1120 FOR I=1 TO M
1130 FOR J=1 TO M
1140 PRINT A(I,J);
1150 NEXT J
1160 PRINT
1170 NEXT I
1180 PRINT
1190 FOR I=1 TO N
1200 FOR J=1 TO N
1210 PRINT Z(I,J);
1220 NEXT J
1230 PRINT
1240 NEXT I
1250 PRINT"SOLUTIONS TO EQUATIONS"
1260 FOR I=1 TO M
1270 P(I)=0
1280 FOR J=1 TO N
1290 P(I)=P(I)+Y(I,J)*H(J)
1300 NEXT J
1310 PRINT I, P(I)
1320 NEXT I
1330 PRINT"NUMBER OF EQUATIONS IS ";N
1340 PRINT"NUMBER OF UNKNOWNS IS ";M
1350 PRINT"CALCULATED AND OBSERVED R.H.S"
1360 FOR I=1 TO N
1370 Q(I)=0
1380 FOR J=1 TO M
1390 Q(I)=Q(I)+X(I,J)*P(J)
1400 NEXT J
1410 PRINT Q(I),H(I),H(I)-Q(I)
1420 NEXT I
1421 PRINT#1
1422 CLOSE1,4
1425 REM
1430 REM ***    TEST DATA    ***
1435 REM
1440 DATA7,13 REM NO OF EQUATIONS AND NO OF UNKNOWN CORRECTIONS
1441 DATA 1,1,1 0,0,0,0,0,0,0,0,0,0,-1.2
1442 DATA 0,-1,0,1,1,0,0,0,0,0,0,0,0,-.5
1443 DATA 0,0,0,0,-1,1,1,0,0,0,0,0,0,1.5
1444 DATA 0,0,0,0,0,-1,0,0,0,0,1,1,0,-2.5
1445 DATA 0,0,0,0,0,0,-1,-1,0,1,0,0,0,-2.3
1446 DATA 0,0,-1,0,0,0,0,1,1,0,0,0,0,1.9
1447 DATA 0,0,0,0,0,0,0,0,0,-1,0,-1,1,1.8
```

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ΠΕΡΙΛΗΨΗ

Μια απλή μέθοδος προσαρμογής δικτύου βάσεων βαρυτομετρικών μετρήσεων με τη χρήση μικροϋπολογιστή

από

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Στην εργασία αυτή παρουσιάζεται μια διαφορετική αντιμετώπιση του προβλήματος της προσαρμογής βαρυτομετρικών δικτύων που βασίζεται στην επίλυση ενός συστήματος γραμμικών εξισώσεων με την μέθοδο της γενικευμένης αναστροφής.

Παρέχεται επίσης ένα πρόγραμμα σε γλώσσα προγραμματισμού BASIC για τον μικροϋπολογιστή Commodore Pet καθώς και ένα παράδειγμα εφαρμογής του.