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SHORT NOTE

A BASIC PROGRAM FOR 2-D SPECTRAL ANALYSIS OF GRAVITY DATA AND SOURCE-DEPTH ESTIMATION

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The spectral methods of analysis have been employed increasingly in recent years. In these methods, the characteristics of the observed anomalies are studied by first transforming the data from the space to the frequency domain and then analyzing their frequency characteristics (Bath, 1974).

The amplitude and phase relationships among the various frequencies has been used extensively by many workers for the interpretation of gravity data, particularly in the situation of downward continuation and source-depth estimation (Spector and Grant, 1970; Treitel, Clement, and Kaul, 1971; Green, 1972; Hahn, Kind, and Mishra, 1976; Pal, Khurana, and Unnikrishnan, 1979; Negi, Agrawal, and Rao, 1983; Bose and Sengupta, 1984; Tselentis, Dimitriadis, and Drakopoulos, 1986).

Following the approaches of Battacharya (1966) and Treitel, Clement, and Kaul (1971), the power spectrum, when amplitude is on a logarithmic scale versus a linear scale for the frequency, may show frequency intervals where the logarithms of the amplitudes may be represented by a linear function of frequency, with amplitudes decreasing with increasing frequencies. The slope of the straightline is proportional to the depth to the top of the body. Thus, if k denotes the wavenumber and $S(k)$ the power spectrum, the depth d to the source can be estimated from the relation $S(k) = f(k)$, by employing the formula:

$$\ln S(k) = -2kd. \qquad (1)$$

It is obvious that the same approach can be followed for the situation of 2-dimensional data by computing the radial spectrum of all the particular waves falling within a certain frequency range as explained next.

METHOD OF ANALYSIS

The gravity field values for a block of $N \times N$ equally spaced (gridded) data, are transformed from the space domain to the frequency domain by means of the 2-dimensional discrete Fast Fourier Transform described by Cooley and Tukey (1965).

The Fourier transform of these data results in a set of real X_R and imaginary X_I amplitudes by which the field values given at the grid points (x, y) can be represented by the sum:

$$g(x, y) = \sum_k \sum_m X_R^K \cos[(2\pi/DX \cdot N)(kx + my)] + X_I^K \sin[(2\pi/DX \cdot N)(kx + my)]$$

where DX is the grid interval.

Equation (2) can be written as follows:

$$g(x, y) = \sum_k \sum_m C_M^K \cdot \cos[(2\pi/DX \cdot N)(kx + my) - P_M^K] \qquad (3)$$

where P is the appropriate phase angle, and

$$C_M^K = [(X_R^K)^2 + (X_I^K)^2]^{1/2}. \qquad (4)$$

It is obvious that each C is the amplitude of a partial field wave with wavelength $DX \cdot N/(k^2 + m^2)$ and frequency $F = (k^2 + m^2)$.

In order to calculate the radial spectrum for each data set we start first by calculating the 2-D power spectrum:

$$SP(I, J) = [XR(I, J)^2 + XI(I, J)^2]$$

where $XR(I, J)$ is the real part and $XI(I, J)$ the imaginary part of the data set at the point I, J.

The radial spectrum is calculated by superposing the 2-D spectrum by a number of omocentric rings with center the point (1, 1) (upper left point of the matrix SP) which is the lowest frequency component of the data set (mean val.), and with radial distances

0.0–0.5:(wavenumber = 0.0 cycl/grid interval)
0.5–1.5:(wavenumber = $1.0/(N \cdot DX)$ cycl/grid interval)
1.5–2.5:(wavenumber = $2.0/(N \cdot DX)$ cycl/grid interval
(Pal, Khurana, and Unnikrishnan, 1979; Hahn, Kind, and Mishra, 1976)

Elements of the matrix with $0.5 < (I^2 + J^2)^{1/2} <= 1.5$ are averaged, and so forth to the Nyquist wavenumber $N/2$ (see Fig. 1).

1393.00	176690.00	23885.00	5963.40	5207.40	3440.10	1977.30
0.	1.	2.	3.	4.	5.	6.
88022.	69674.1	492.6	2003.3	887.7	557.9	226.5
1.	1.4	2.23	3.16	4.12	5.09	6.08
31928.	13776.7	1532.6	22.2	552.1	48.5	[illegible]
2.	2.23	2.82	3.6	4.47	5.38	6.32
5343.7	1817.4	[illegible]6.7	222.5	8.4	4.4	12.2
3.	3.16	3.6	4.24	5.	5.83	6.7
5389.8	1313.2	599.1	77.1	23.8	22.6	20.1
4.	4.12	4.47	5.	5.65	6.4	7.21
5300.8	756.5	171.6	8.6	14.5	6.8	7.7
5.	5.1	5.38	5.83	6.4	7.07	7.81
3537.0	712.8	70.7	9.8	10.2	3.7	5.2
6.	6.08	6.32	6.7	7.21	7.81	8.48

Figure 1. 2-D power spectrum and corresponding radial distances for each point.

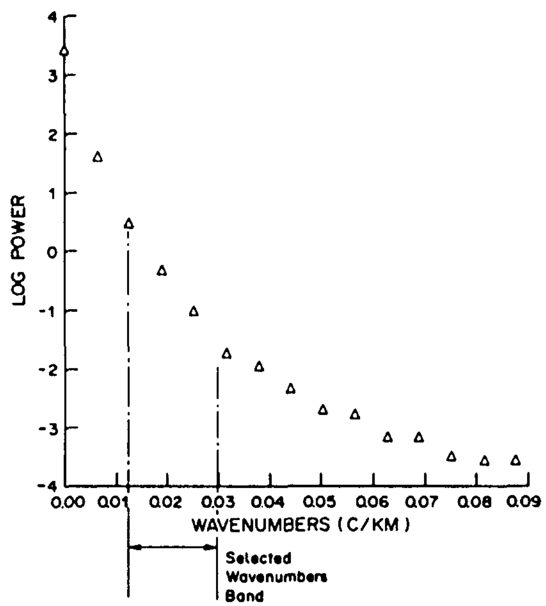


Figure 2. Radial spectrum of masterfile.

After this, the logarithm is taken. The resulting values, form the radial spectrum of the anomalous field under consideration. By this technique we transfer our 2-dimensional problem to a 1-dimensional one.

DESCRIPTION OF THE PROGRAM

The computer program presented in Appendix 2 provides the facility to perform the analytical processes for radial spectra evaluation, automatically for any user-defined data block (window) retrieved from a masterfile (e.g. a digitized map).

The program is written in standard IBM PC BASIC, and there should be little difficulty in running the program on another machine. Difficulties may arise in the plotting subroutine, which either must be adjusted according to the requirements of the system used, or deleted with the spectrum being plotted manually [$B(TC, K) = f(WV(K))$] in order to determine the upper and lower limits for the regression.

Care has to be taken to minimize the aliasing

Masterfile (32X32) Point	Wavenumber (cycl/Km)	Windows (16X16) Point	Wavenumber (cycl/Km)
1	0	1	0
2	.00625	2	.0125
3	.0125	3	.025
4	.01875	4	.0375
5	.025	5	.05
6	.03125	6	.0625
7	.0375	7	.075
8	.04375	8	.0875
9	.05		
10	.05625		
11	.0625		
12	.06875		
13	.075		
14	.08125		
15	.0875		
16	.09375		

Figure 3. Selection of wavenumbers band.

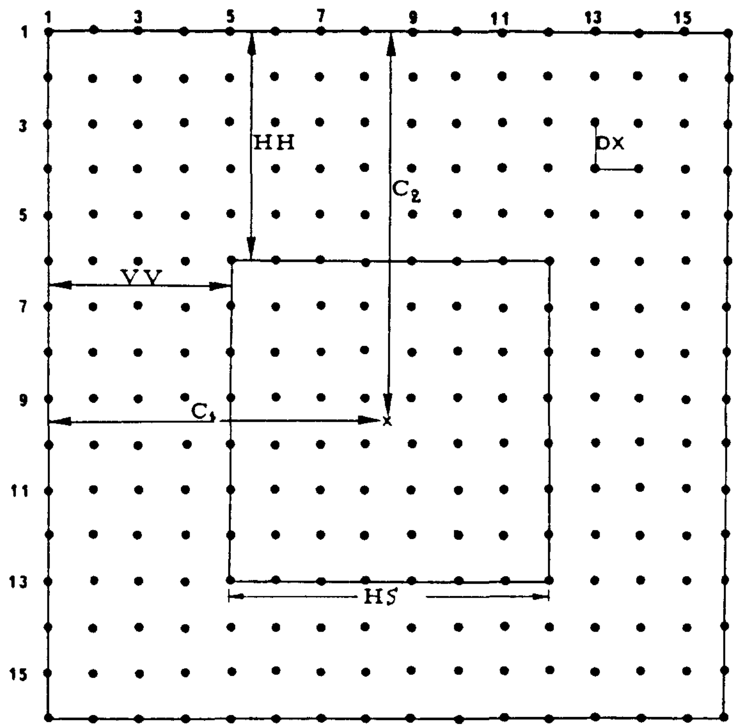


Figure 4. Scanning parameters.

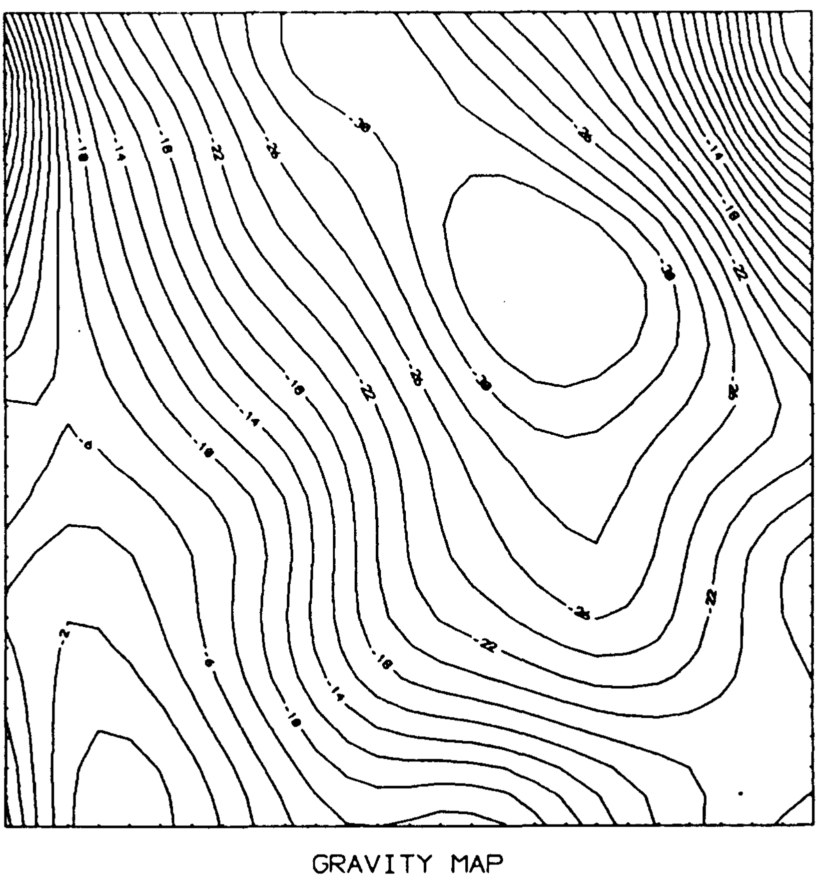


Figure 5. Contour map of masterfile.

Table 1. Data set used in sample run

-29	-30	-31	-31	-31	-32	-34	-38	-41	-43	-44	-43	-40	-35	-29	-21	-17	-15	-16	-18
-19	-16	-12	-6	4	16	30	42	53	62	72	85								
-27	-27	-27	-27	-28	-30	-33	-38	-43	-46	-47	-46	-43	-40	-35	-30	-25	-22	-21	-21
-20	-18	-15	-9	0	13	27	40	51	60	70	82								
-24	-24	-23	-23	-23	-26	-31	-37	-42	-47	-47	-45	-43	-42	-39	-35	-31	-27	-24	-22
-21	-19	-17	-11	-2	10	25	39	50	59	68	78								
-20	-19	-18	-18	-19	-22	-26	-33	-38	-40	-41	-40	-40	-40	-39	-37	-34	-31	-27	-24
-22	-21	-19	-12	-5	7	23	37	47	55	64	73								
-17	-16	-15	-15	-16	-19	-24	-30	-34	-36	-36	-35	-35	-37	-38	-38	-36	-33	-29	-26
-24	-23	-22	-18	-10	3	18	32	42	48	58	66								
-15	-14	-13	-14	-15	-18	-23	-28	-32	-33	-32	-31	-32	-35	-38	-39	-39	-36	-33	-29
-27	-26	-25	-23	-16	-3	11	24	33	41	48	57								
-13	-12	-12	-13	-15	-20	-25	-29	-32	-32	-31	-31	-32	-35	-39	-42	-43	-42	-39	-35
-32	-32	-31	-28	-22	-10	4	16	24	30	38	48								
-11	-10	-11	-13	-16	-21	-26	-30	-32	-32	-31	-31	-33	-37	-41	-46	-49	-48	-46	-42
-40	-38	-37	-34	-27	-16	-3	8	15	21	30	42								
-9	-9	-9	-12	-16	-21	-25	-29	-31	-31	-30	-30	-32	-37	-42	-48	-51	-52	-51	-49
-46	-44	-43	-39	-32	-22	-8	2	9	15	25	38								
-8	-7	-8	-11	-15	-20	-24	-27	-28	-29	-28	-28	-30	-35	-42	-47	-52	-54	-54	-52
-50	-48	-47	-43	-39	-25	-13	-3	4	11	22	36								
-6	-6	-7	-9	-13	-17	-21	-24	-25	-25	-25	-25	-28	-34	-40	-46	-51	-53	-54	-53
-51	-50	-48	-45	-39	-30	-18	-8	-1	7	18	33								
-5	-4	-5	-7	-11	-15	-19	-21	-22	-23	-22	-23	-27	-32	-38	-43	-49	-51	-52	-52
50	-48	-47	-44	-40	-32	-22	-13	-6	1	13	27								
-3	-3	-4	-6	-9	-13	-17	-19	-20	-21	-21	-22	-26	-30	-36	-41	-46	-48	-50	-49
-47	-45	-43	-41	-39	-33	-26	-18	-12	-6	6	20								
-2	-2	-3	-5	-8	-12	-15	-18	-19	-19	-20	-21	-24	-29	-34	-39	-43	-45	-47	-46
-43	-41	-39	-37	-36	-33	-28	-23	-19	-13	-3	11								
-0	-1	-2	-4	-7	-10	-15	-15	-16	-17	-18	-19	-22	-26	-31	-35	-39	-43	-43	-42
-39	-37	-34	-33	-33	-32	-30	-28	-24	-19	-10	1								
1	-0	-1	-3	-6	-8	-10	-12	-13	-14	-15	-17	-21	-25	-29	-32	-35	-37	-39	-39
-37	-34	-31	-29	-30	-30	-30	-29	-27	-23	-18	-6								
2	1	-0	-2	-3	-5	-6	-8	-8	-10	-14	-17	-21	-24	-27	-29	-32	-34	-37	-37
-36	-33	-29	-27	-27	-28	-28	-28	-26	-24	-19	-10								
3	2	1	-0	-1	-1	-2	-2	-3	-7	-12	-17	-22	-26	-28	-29	-30	-33	-35	-37
-36	-34	-31	-26	-24	-23	-23	-23	-22	-20	-15	-8								
4	3	3	2	1	1	2	3	1	-4	-11	-18	-24	-28	-29	-29	-30	-32	-35	-38
-38	-36	-30	-25	-22	-19	-17	-17	-14	-12	-9	-5								
5	4	4	3	2	2	4	4	2	-3	-11	-20	-27	-30	-30	-30	-30	-32	-35	-39
-39	-37	-31	-25	-20	-15	-13	-10	-6	-5	-4	-1								
6	5	5	3	2	2	3	3	0	-5	-13	-22	-28	-31	-32	-30	-30	-32	-35	-39
-40	-38	-33	-26	-20	-14	-10	-8	-4	-0	2	3								
6	6	4	3	1	-0	-0	-1	-4	-9	-17	-24	-30	-32	-34	-32	-30	-31	-35	-38
-40	-39	-34	-30	-22	-16	-12	-9	-5	-2	1	1								
7	6	5	3	0	-2	-4	-6	-9	-14	-20	-26	-30	-32	-31	-29	-28	-30	-33	-36
-38	-37	-34	-30	-24	-18	-14	-11	-8	-7	-7	-7								
7	7	6	4	1	-3	-5	-8	-11	-16	-21	-25	-29	-30	-29	-27	-27	-28	-31	-33
-35	-35	-34	-30	-25	-20	-16	-14	-12	-10	-10	-10								
7	8	7	5	2	-1	-4	-8	-12	-16	-20	-24	-27	-27	-26	-25	-24	-26	-28	-30
-32	-33	-31	-28	-25	-21	-18	-16	-14	-13	-13	-14								
8	9	9	7	5	1	-3	-6	-10	-13	-17	-21	-23	-23	-22	-21	-21	-22	-23	-25
-27	-28	-26	-24	-22	-19	-17	-16	-15	-15	-16	-17								
9	10	10	9	7	4	0	-3	-6	-10	-14	-16	-18	-18	-17	-15	-15	-16	-18	-21
-21	-21	-21	-20	-19	-18	-17	-17	-17	-17	-19	-20								
10	11	11	10	9	6	3	-1	-4	-7	-11	-13	-14	-13	-10	-8	-8	-9	-12	-14
-16	-16	-16	-16	-17	-16	-17	-18	-19	-20	-22	-23								
11	12	12	11	10	8	5	2	-2	-5	-8	-9	-9	-6	-2	-0	1	-2	-5	-7
-9	-11	-13	-14	-16	-16	-17	-19	-21	-23	-24	-25								
12	13	13	13	11	9	6	3	1	-3	-5	-6	-4	-0	5	9	8	5	1	-3
-6	-9	-12	-14	-17	-18	-20	-22	-24	-26	-28	-28								
14	14	14	14	12	10	8	5	2	-1	-3	-3	-0	4	10	13	12	9	3	-2
-7	-11	-15	-17	-21	-23	-24	-26	-28	-30	-32	-32								
16	16	16	15	13	11	9	6	4	1	-1	-1	2	8	13	15	12	7	1	-6
-11	-16	-20	-23	-25	-27	-28	-29	-31	-33	-35	-35								

errors caused by digitization and to avoid the truncated anomalies.

The errors in source-depth prediction are related closely to the window size. For gravity fields, a window of six times the source depth is required for < 10% information loss (Regan and Hinze, 1976).

In this sample run, the window has the size of 80 × 80 km (16 × 16, grid interval 5 km).

Adopting this selection, source depths up to 15 km are valid.

Care also has to be taken in the selection of the linear segments. The program starts by computing the radial spectrum of the masterfile. This way all the frequency components and their corresponding wavenumbers are presented on the screen so the user has only to select the proper linear segments (see Figs. 2 and 3).

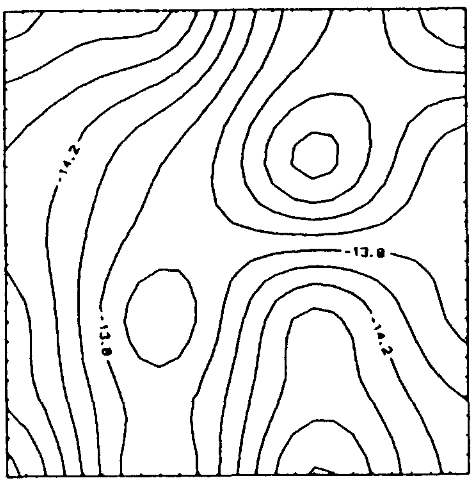
After defining the size and location (Fig. 4) of the first window, the program automatically scans over at the predetermined step interval. The depth to the

Table 2. Sample run

Input parameters
Grid spacing in km:5.0
Window size (power of two):16
Input filename:sample dat
No. of columns in the masterfile:32
No. of rows:32
Horizontal overlapping step:4
Vertical overlapping step:4
Horiz. coord. to start the scan:1
Vertical coord. to start the scan:1
Left regression point:2
Right regression point:4

source for the specific window and the corresponding rms error is evaluated.

Another feature of the program is that it plots the radial spectra of each window used, so the user can verify the previous selection of the upper and lower limits in the spectrum graph. (It is obvious that the various sections of the spectrum correspond to different source depths.)



SOURCE DEPTHS

Figure 6. Contour map of source depths.

EXAMPLE

In this sample run, we use a master file (Tables 1 and 2) consisting of 32 × 32 data points with grid spacing 5 km. This file represents the total Bouguer anomaly in northwestern Greece (Fig. 5).

The same discontinuity has been revealed by Makris (1977) employing seismic refraction methods.

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APPENDIX 1

Listing of Computer Program

```
******** PRELIMINARY RUN ********
Direct Transform ..

wvnbr 0.0063 c/Km    Spectr  3.421
wvnbr 0.0125 c/Km    Spectr  1.624
wvnbr 0.0188 c/Km    Spectr  0.535
wvnbr 0.0250 c/Km    Spectr -0.159
wvnbr 0.0313 c/Km    Spectr -0.882
wvnbr 0.0375 c/Km    Spectr -1.610
wvnbr 0.0437 c/Km    Spectr -1.832
wvnbr 0.0500 c/Km    Spectr -2.173
wvnbr 0.0562 c/Km    Spectr -2.563
wvnbr 0.0625 c/Km    Spectr -2.700
```

```
wvnbr 0.0688 c/Km   Spectr -3.016
wvnbr 0.0750 c/Km   Spectr -3.049
wvnbr 0.0812 c/Km   Spectr -3.372
wvnbr 0.0875 c/Km   Spectr -3.412
wvnbr 0.0938 c/Km   Spectr -3.401
  - Window center at the grid point: 9     9
Direct Transform ..


EVALUATING SPECTRUM
wvnbr 0.0125 c/Km   Spectr  2.609
wvnbr 0.0250 c/Km   Spectr  1.312
wvnbr 0.0375 c/Km   Spectr  0.107
wvnbr 0.0500 c/Km   Spectr -1.109
wvnbr 0.0625 c/Km   Spectr -1.423
wvnbr 0.0750 c/Km   Spectr -1.954
wvnbr 0.0875 c/Km   Spectr -2.062
Slope: -96.85823    Slope st.error:  .2974908
Depth:  15.41548    Depth st.error: 4.734717E-02
--------------------------------------------------------
  - Window center at the grid point: 9     13
Direct Transform ..


EVALUATING SPECTRUM
wvnbr 0.0125 c/Km   Spectr  2.131
wvnbr 0.0250 c/Km   Spectr  0.890
wvnbr 0.0375 c/Km   Spectr -0.343
wvnbr 0.0500 c/Km   Spectr -1.563
wvnbr 0.0625 c/Km   Spectr -1.819
wvnbr 0.0750 c/Km   Spectr -2.384
wvnbr 0.0875 c/Km   Spectr -2.472
Slope: -98.15307    Slope st.error:  .3221173
Depth:  15.62156    Depth st.error: .0512666
--------------------------------------------------------
  - Window center at the grid point: 9     17
Direct Transform ..


EVALUATING SPECTRUM
wvnbr 0.0125 c/Km   Spectr  2.217
wvnbr 0.0250 c/Km   Spectr -0.105
wvnbr 0.0375 c/Km   Spectr -1.100
wvnbr 0.0500 c/Km   Spectr -2.196
wvnbr 0.0625 c/Km   Spectr -2.636
wvnbr 0.0750 c/Km   Spectr -3.114
wvnbr 0.0875 c/Km   Spectr -3.154
Slope: -83.66921    Slope st.error:  2.350898
Depth:  13.31638    Depth st.error: .3741573
--------------------------------------------------------
  - Window center at the grid point: 9     21
Direct Transform ..


EVALUATING SPECTRUM
wvnbr 0.0125 c/Km   Spectr  3.569
wvnbr 0.0250 c/Km   Spectr  1.616
wvnbr 0.0375 c/Km   Spectr  0.822
wvnbr 0.0500 c/Km   Spectr -0.267
wvnbr 0.0625 c/Km   Spectr -0.603
wvnbr 0.0750 c/Km   Spectr -1.135
wvnbr 0.0875 c/Km   Spectr -1.244
Slope: -75.32616    Slope st.error:  6.836991
Depth:  11.98854    Depth st.error: 1.088142
--------------------------------------------------------
  - Window center at the grid point: 13    9
Direct Transform ..


EVALUATING SPECTRUM
wvnbr 0.0125 c/Km   Spectr  2.853
wvnbr 0.0250 c/Km   Spectr  1.373
wvnbr 0.0375 c/Km   Spectr  0.165
wvnbr 0.0500 c/Km   Spectr -0.907
wvnbr 0.0625 c/Km   Spectr -1.195
wvnbr 0.0750 c/Km   Spectr -1.663
wvnbr 0.0875 c/Km   Spectr -1.807
Slope: -91.17591    Slope st.error:  3.146165
Depth:  14.51111    Depth st.error: .5007282
--------------------------------------------------------
  - Window center at the grid point: 13    13
Direct Transform ..
```

```
EVALUATING SPECTRUM
wvnbr 0.0125 c/Km   Spectr  3.059
wvnbr 0.0250 c/Km   Spectr  0.998
wvnbr 0.0375 c/Km   Spectr -0.084
wvnbr 0.0500 c/Km   Spectr -1.142
wvnbr 0.0625 c/Km   Spectr -1.374
wvnbr 0.0750 c/Km   Spectr -1.927
wvnbr 0.0875 c/Km   Spectr -2.003
Slope: -85.61389    Slope st.error:  .568758
Depth:  13.62589    Depth st.error: 9.052073E-02
-------------------------------------------------------
  - Window center at the grid point: 13    17
Direct Transform ..


EVALUATING SPECTRUM
wvnbr 0.0125 c/Km   Spectr  2.764
wvnbr 0.0250 c/Km   Spectr -0.336
wvnbr 0.0375 c/Km   Spectr -1.288
wvnbr 0.0500 c/Km   Spectr -2.109
wvnbr 0.0625 c/Km   Spectr -2.613
wvnbr 0.0750 c/Km   Spectr -3.236
wvnbr 0.0875 c/Km   Spectr -3.268
Slope: -70.90147    Slope st.error:  3.008824
Depth:  11.28433    Depth st.error: .4788696
-------------------------------------------------------
  - Window center at the grid point: 13    21
Direct Transform ..


EVALUATING SPECTRUM
wvnbr 0.0125 c/Km   Spectr  2.941
wvnbr 0.0250 c/Km   Spectr  0.783
wvnbr 0.0375 c/Km   Spectr -0.270
wvnbr 0.0500 c/Km   Spectr -1.457
wvnbr 0.0625 c/Km   Spectr -1.804
wvnbr 0.0750 c/Km   Spectr -2.398
wvnbr 0.0875 c/Km   Spectr -2.485
Slope: -89.60703    Slope st.error:  3.088035
Depth:  14.26141    Depth st.error: .4914765
-------------------------------------------------------
  - Window center at the grid point: 17    9
Direct Transform ..


EVALUATING SPECTRUM
wvnbr 0.0125 c/Km   Spectr  2.988
wvnbr 0.0250 c/Km   Spectr  1.356
wvnbr 0.0375 c/Km   Spectr  0.047
wvnbr 0.0500 c/Km   Spectr -1.007
wvnbr 0.0625 c/Km   Spectr -1.273
wvnbr 0.0750 c/Km   Spectr -1.746
wvnbr 0.0875 c/Km   Spectr -1.896
Slope: -94.52674    Slope st.error:  5.880555
Depth:  15.04441    Depth st.error: .9359201
-------------------------------------------------------
  - Window center at the grid point: 17    13
Direct Transform ..


EVALUATING SPECTRUM
wvnbr 0.0125 c/Km   Spectr  3.247
wvnbr 0.0250 c/Km   Spectr  1.029
wvnbr 0.0375 c/Km   Spectr  0.180
wvnbr 0.0500 c/Km   Spectr -0.909
wvnbr 0.0625 c/Km   Spectr -1.141
wvnbr 0.0750 c/Km   Spectr -1.676
wvnbr 0.0875 c/Km   Spectr -1.781
Slope: -77.50523    Slope st.error:  5.516734
Depth:  12.33535    Depth st.error: .8780162
-------------------------------------------------------
  - Window center at the grid point: 17    17
Direct Transform ..


EVALUATING SPECTRUM
wvnbr 0.0125 c/Km   Spectr  2.858
wvnbr 0.0250 c/Km   Spectr  0.558
wvnbr 0.0375 c/Km   Spectr -0.942
wvnbr 0.0500 c/Km   Spectr -1.914
wvnbr 0.0625 c/Km   Spectr -2.234
wvnbr 0.0750 c/Km   Spectr -2.838
wvnbr 0.0875 c/Km   Spectr -2.860
```

```
Slope: -98.86496     Slope st.error:  12.20325
Depth:  15.73486     Depth st.error: 1.94221
-------------------------------------------------------
  - Window center at the grid point: 17    21
Direct Transform ..


EVALUATING SPECTRUM
wvnbr 0.0125 c/Km    Spectr  2.567
wvnbr 0.0250 c/Km    Spectr -0.300
wvnbr 0.0375 c/Km    Spectr -1.015
wvnbr 0.0500 c/Km    Spectr -2.395
wvnbr 0.0625 c/Km    Spectr -2.752
wvnbr 0.0750 c/Km    Spectr -3.480
wvnbr 0.0875 c/Km    Spectr -3.526
Slope: -83.76724     Slope st.error:  15.3509
Depth:  13.33198     Depth st.error: 2.443174
-------------------------------------------------------
  - Window center at the grid point: 21    9
Direct Transform ..


EVALUATING SPECTRUM
wvnbr 0.0125 c/Km    Spectr  2.882
wvnbr 0.0250 c/Km    Spectr  1.363
wvnbr 0.0375 c/Km    Spectr -0.106
wvnbr 0.0500 c/Km    Spectr -1.099
wvnbr 0.0625 c/Km    Spectr -1.354
wvnbr 0.0750 c/Km    Spectr -1.869
wvnbr 0.0875 c/Km    Spectr -1.962
Slope: -98.4774     Slope st.error:  10.98003
Depth:  15.67318     Depth st.error: 1.747527
-------------------------------------------------------
  - Window center at the grid point: 21    13
Direct Transform ..


EVALUATING SPECTRUM
wvnbr 0.0125 c/Km    Spectr  2.893
wvnbr 0.0250 c/Km    Spectr  1.260
wvnbr 0.0375 c/Km    Spectr  0.388
wvnbr 0.0500 c/Km    Spectr -0.742
wvnbr 0.0625 c/Km    Spectr -1.001
wvnbr 0.0750 c/Km    Spectr -1.540
wvnbr 0.0875 c/Km    Spectr -1.604
Slope: -80.09995     Slope st.error:  5.926948
Depth:  12.74831     Depth st.error: .9433038
-------------------------------------------------------
  - Window center at the grid point: 21    17
Direct Transform ..


EVALUATING SPECTRUM
wvnbr 0.0125 c/Km    Spectr  2.164
wvnbr 0.0250 c/Km    Spectr  1.280
wvnbr 0.0375 c/Km    Spectr -0.122
wvnbr 0.0500 c/Km    Spectr -1.214
wvnbr 0.0625 c/Km    Spectr -1.517
wvnbr 0.0750 c/Km    Spectr -2.043
wvnbr 0.0875 c/Km    Spectr -2.102
Slope: -99.73073     Slope st.error:  7.157816
Depth:  15.87265     Depth st.error: 1.139203
-------------------------------------------------------
  - Window center at the grid point: 21    21
Direct Transform ..


EVALUATING SPECTRUM
wvnbr 0.0125 c/Km    Spectr  1.714
wvnbr 0.0250 c/Km    Spectr  0.768
wvnbr 0.0375 c/Km    Spectr -0.297
wvnbr 0.0500 c/Km    Spectr -1.438
wvnbr 0.0625 c/Km    Spectr -1.811
wvnbr 0.0750 c/Km    Spectr -2.323
wvnbr 0.0875 c/Km    Spectr -2.359
Slope: -88.23514     Slope st.error:  1.782593
Depth:  14.04307     Depth st.error: .2837087
-------------------------------------------------------
```

APPENDIX 2

```
1000 REM ********************************************
1010 REM *              R A D F R E Q               *
1020 REM * K.DIMITRIADIS. G-A.TSELENTIS. C.THANASSOULAS *
1030 REM * This program estimates source depths to   *
1040 REM * basement rocks using the concept of radial *
1050 REM * frequency.                                *
1060 REM *-SUBROUTINES USED: -1DFFT                  *
1070 REM *                   -2DFFT                  *
1080 REM *                   -ROWS & COLUMNS         *
1090 REM *                   -WINDOW                 *
1100 REM *                   -SCAN                   *
1110 REM ********************************************
1120 DIM XR(64),XI(64),WV(32)
1130 DIM IR(32),CO(32),SI(32),B(50,16)
1140 DIM RM(64,64),IM(64,64),A(64,64)
1150 REM
1160 CLS
1170 TC=0: REM Counter for each block
1180 POKE 49152!,0
1190 INPUT"Enter grid spacing in Km";DX
1200 INPUT"Enter window size (power of two)";HS
1210 INPUT"Enter input file name ";FI$
1220 VS=HS:REM Horizontal=Vertical size
1230 INPUT"Enter number of rows in the masterfile";RO
1240 INPUT"Enter number of columns ";CL
1250 REM * Input the data file *
1260 OPEN FI$ FOR INPUT AS #2
1270 FOR I=1 TO RO
1280 FOR J=1 TO CL
1290 INPUT#2,A(I,J)
1300 NEXT J
1310 NEXT I
1320 CLOSE #2
1330 GOSUB 3600:REM FIRST RUN
1340 X1=VS:Y1=VS:GOSUB 3350:REM GENERATE WAVENUMBERS
1350 REM *** SCAN PARAMETERS ***
1360 INPUT"Horiz overlapping step";OP
1370 INPUT"Vertical              ";VO
1380 INPUT"Horiz coord to start the scan";HC
1390 INPUT"Vertical                     ";VC
1400 INPUT"Left regression point";LL
1410 INPUT"Right                ";UP
1420 FOR HH=VC TO RO STEP VO
1430 FOR VV=HC TO CL STEP OP
1440 IF ((HH+VS>RO) OR (VV+VS>CL)) THEN 1730
1450 C2=(X1/2)+VV
1460 C1=(X1/2)+HH
1470 LPRINT"  - Window center at the grid point:";C1,C2
1480 GOSUB 3180:REM TAKE ONE WINDOW FROM MASTERFILE
1490 LX=X1/2:LY=Y1/2
1500 SG=-1:GOSUB 1760:REM SG=1 FOR INVERSE TRANSFORM
1510 LPRINT"EVALUATING SPECTRUM"
1520 FOR I=1 TO LX
1530 FOR J=1 TO LY
1540 REM COMPUTE THE POWER SPECTRUM [PS=(Real^2+Imagin^2)]
1550 B(I,J)=(RM(I,J)*RM(I,J)+IM(I,J)*IM(I,J))
1560 NEXT J
1570 NEXT I
1580 FOR I=1 TO LX
1590 FOR J=1 TO LY
1600 RM(I,J)=B(I,J)
1610 NEXT J
1620 NEXT I
1630 REM ** ZERO THE B ARRAY
1640 REM ** FOR LATER USE
1650 FOR I=1 TO LX
1660 FOR J=1 TO LY
1670 B(I,J)=0!
1680 NEXT J
1690 NEXT I
1700 GOSUB 2860
1710 GOSUB 3800
1720 GOSUB 3410
1730 NEXT VV
1740 NEXT HH
1750 END
1760 REM *** 2D FFT *ROUTINES DRIVER
1770 IF SG=1 THEN LPRINT"Inverse Transform .."
1780 IF SG=-1 THEN LPRINT"Direct Transform .."
1790 REM ** PROCEDURE ROWS AND COLUMNS **
1800 REM ** ASSIGN PARAMETERS FOR FFT
```

```
1810 REM ** ALGORITHM
1820 POKE 49152!,0
1830 N=Y1:N2=Y1/2:N3=N2+1:RN=N:R3=N2
1840 K=LOG(N)/LOG(2)
1850 REM
1860 FOR O=1 TO X1:REM ROWS
1870 FOR S=1 TO Y1
1880 XR(S)=RM(O,S)
1890 XI(S)=IM(O,S)
1900 NEXT S
1910 GOSUB 2170
1920 REM
1930 FOR S=1 TO N
1940 RM(O,S)=XR(S)
1950 IM(O,S)=XI(S)
1960 NEXT S
1970 NEXT O
1980 LPRINT
1990 N=X1:N2=X1/2:N3=N2+1:RN=N:R3=N2
2000 K=LOG(N)/LOG(2)
2010 POKE 49152!,0
2020 FOR O=1 TO Y1:REM COLUMNS
2030 FOR S=1 TO X1
2040 XR(S)=RM(S,O)
2050 XI(S)=IM(S,O)
2060 NEXT S
2070 GOSUB 2170
2080 FOR S=1 TO N
2090 RM(S,O)=XR(S)
2100 IM(S,O)=XI(S)
2110 NEXT S
2120 NEXT O
2130 LPRINT:PRINT
2140 RETURN
2150 REM
2160 REM
2170 REM ** PROCEDURE 1DFFT **
2180 REM ***************************************************
2190 REM * THIS PROCEDURE USES THE ALGORITHM OF COOLEY & *
2200 REM * TUKEY, TO PERFORM THE FOURIER TRANSFORM.       *
2210 REM *   XR AND XI ARE THE REAL AND IMAGINARY PARTS. *
2220 REM ***************************************************
2230 REM
2240 IF PEEK(49152!)=211 THEN 2480
2250 POKE 49152!,211
2260 REM * 49152 ADDRESS IS A RAM ADDR OF THE SYSTEM
2270 REM * IN USE, AND THE VALUE 1 IS USED AS STATUS INDEX
2280 REM * TO AVOID THE CONTINUUS COMPUTATION OF THE SINUS
2290 REM * AND COSINUS .
2300 REM
2310 REM
2320 IR(1)=0
2330 FOR J=1 TO K
2340 ID=2^(J-1)
2350 FOR I=1 TO ID
2360 IR(I)=IR(I)*2
2370 IF J<K THEN IR(I+ID)=IR(I)+1
2380 NEXT I
2390 NEXT J
2400 F1=N
2410 W=2*3.14159/F1
2420 FOR I=1 TO ID
2430 FI=IR(I)/2
2440 A=FI*W
2450 CO(I)=COS(A)
2460 SI(I)=SIN(A)
2470 NEXT I
2480 FOR NC=1 TO K
2490 NB=2^(NC-1)
2500 LB=N/NB
2510 L2=LB/2
2520 FOR IB=1 TO NB
2530 C=CO(IB)
2540 S=SG*SI(IB)
2550 IS=(IB-1)*LB+1
2560 FF=(IB-1)*LB+L2
2570 FOR I=IS TO FF
2580 I2=I+L2
2590 QR=XR(I2)*C-XI(I2)*S
2600 QI=XR(I2)*S+XI(I2)*C
2610 XR(I2)=XR(I)-QR
2620 XI(I2)=XI(I)-QI
2630 XR(I)=XR(I)+QR
2640 XI(I)=XI(I)+QI
```

```
2650 NEXT I
2660 NEXT IB
2670 NEXT NC
2680 FOR I=1 TO ID
2690 FOR L=1 TO 2
2700 ER=IR(I)+L
2710 II=I+(L-1)*ID
2720 IF ER<II THEN 2790
2730 ZR=XR(ER)
2740 ZI=XI(ER)
2750 XR(ER)=XR(II)
2760 XI(ER)=XI(II)
2770 XR(II)=ZR
2780 XI(II)=ZI
2790 NEXT L
2800 NEXT I
2810 IF SG>0 THEN RETURN
2820 FOR I=1 TO N
2830 XR(I)=XR(I)/N:XI(I)=XI(I)/N
2840 NEXT I
2850 RETURN
2860 REM ****************************************************
2870 REM * THIS PROCEDURE COMPUTES THE RADIAL SPECTRUM FROM    *
2880 REM * THE 2 - D SPECTRUM PRODUCED BY THE FFT.            *
2890 REM *   FOR THIS. A RADIAL SCANNING IS PERFORMED,USING   *
2900 REM * AS CENTER THE POINT  (1,1)  . TO FIND THE POINTS   *
2910 REM * LUYING INSIDE CONSECUTIVE RINGS WITH RADIUS:       *
2920 REM *                     0.0 - 0.5                      *
2930 REM *                     0.5 - 1.5    ETC.              *
2940 REM ****************************************************
2950 TC=TC+1
2960 FOR K=1 TO LX-1
2970 B(TC,K)=0
2980 CN=0
2990 HI=K+.5
3000 LO=K-.5
3010 FOR I=1 TO K+1
3020 FOR J=1 TO K+1
3030 RDI=I-1
3040 RDJ=J-1
3050 DI=SQR(RDI*RDI+RDJ*RDJ)
3060 IF((DI<LO) OR (DI>=HI)) GOTO 3090
3070 B(TC,K)=B(TC,K)+RM(I,J)
3080 CN=CN+1
3090 NEXT J
3100 NEXT I
3110 B(TC,K)=B(TC,K)/CN
3120 REM *** THE POINTS ARE AVERAGED **
3130 B(TC,0)=0:REM VALUE AT ZERO WAVENUMBER
3140 B(TC,K)=LOG(B(TC,K))
3150 LPRINT USING"wvnbr #.#### c/Km";WV(K);:LPRINT USING"   Spectr ##.###";
B(TC,K)
3160 NEXT K
3170 RETURN
3180 REM ** PROCEDURE WINDOW **
3190 REM ****************************************************
3200 REM * THIS PROCEDURE TAKES ONE BLOCK FROM THE MASTERFILE *
3210 REM * ACCORDING TO THE DEFINED PARAMETERS.               *
3220 REM ****************************************************
3230 REM
3240 REM
3250 FOR I=1 TO RO
3260 FOR J=1 TO CL
3270 IF (I>HH+VS AND J>VV+HS) THEN 3340
3280 IF ((J<VV)OR(J>=VV+HS)) THEN 3320
3290 IF ((I<HH)OR(I>=HH+VS)) THEN 3320
3300 II=I-HH+1:JJ=J-VV+1
3310 RM(II,JJ)=A(I,J)
3320 NEXT J
3330 NEXT I
3340 RETURN
3350 REM **** WAVENUMBERS ****
3360 FOR U=1 TO X1/2
3370 WV(U)=U/(DX*X1)
3380 NEXT U
3390 WV(0)=0
3400 RETURN
3410 REM ********* LINEFIT ***********
3420 SU=0:SX=0:SY=0:UY=0:XY=0
3430 NN=UP-LL+1
3440 FOR J=LL TO UP
3450 XD=WV(J):YD=B(TC,J)
3460 SU=SU+XD
3470 UY=UY+YD
```

```
3480 SX=SX+XD*XD
3490 SY=SY+YD*YD
3500 XY=XY+XD*YD
3510 NEXT J
3520 SL=(NN*XY-SU*UY)/(NN*SX-SU*SU)
3530 B=((SX*UY-SU*XY)/(NN*SX-SU*SU))
3540 VR=(SY+B*B*NN+SL*SL*SX-2*(B*UY+SL*XY-B*SL*SU))/(NN-2)
3550 SP=SQR(NN*VR/(NN*SX-SU*SU))
3560 DP=-SL/(2*3.14159)
3570 LPRINT"Slope: ";SL;"   Slope st.error: ";SP
3580 LPRINT"Depth: ";DP;"   Depth st.error:";SP/(2*3.14159)
3585 LPRINT"-------------------------------------------------------------"
3590 RETURN
3600 REM FIRST RUN
3610 LPRINT" ******* PRELIMINARY RUN ******* "
3620 X1=RO:Y1=CL:LX=X1/2:LY=Y1/2
3630 FOR I=1 TO X1:FOR J=1 TO Y1:RM(I,J)=A(I,J)
3640 NEXT J:NEXT I
3650 SG=-1:GOSUB 1760:GOSUB 3350
3660 FOR I=1 TO LX:FOR J=1 TO LY
3670 B(I,J)=RM(I,J)*RM(I,J)+IM(I,J)*IM(I,J)
3680 NEXT J:NEXT I
3690 FOR I=1 TO LX:FOR J=1 TO LY
3700 RM(I,J)=B(I,J)
3710 NEXT J:NEXT I
3720 GOSUB 2860:GOSUB 3800
3730 RETURN
3800 REM ************************************************
3810 REM * THIS SUBROUTINE PLOTS THE B(tc,k) = f(wv(k))  *
3820 REM ************************************************
3830 FA=16
3840 REM
3850 SCREEN 1
3860 KEY OFF
3870 LINE (50,10) - (50,180)
3880 LINE (50,120) - (320,120)
3890 PRINT"              Radial spectrum"
3895 PRINT"              Window: ";TC
3900 FOR I=1 TO LX-1
3910 LINE (54,120-I*FA) - (50,120-I*FA)
3920 LINE (50+I*25,120) - (50+I*25,124)
3930 NEXT I
3940 PSET(50,120-B(TC,1)*FA)
3950 FOR I=1 TO LX-2
3960 LINE (25+I*25,120-B(TC,I)*FA) - (25+(I+1)*25,120-FA*B(TC,I+1))
3970 NEXT I
3980 IF INKEY$="" THEN 3980
3990 SCREEN 2,0,0:SCREEN 0,0,0
4000 RETURN
```